

An illustration of a teal pipe with blue flanges. A small hole in the pipe is leaking a dark purple liquid, with a single drop falling below it.

Eww You're Leaking:

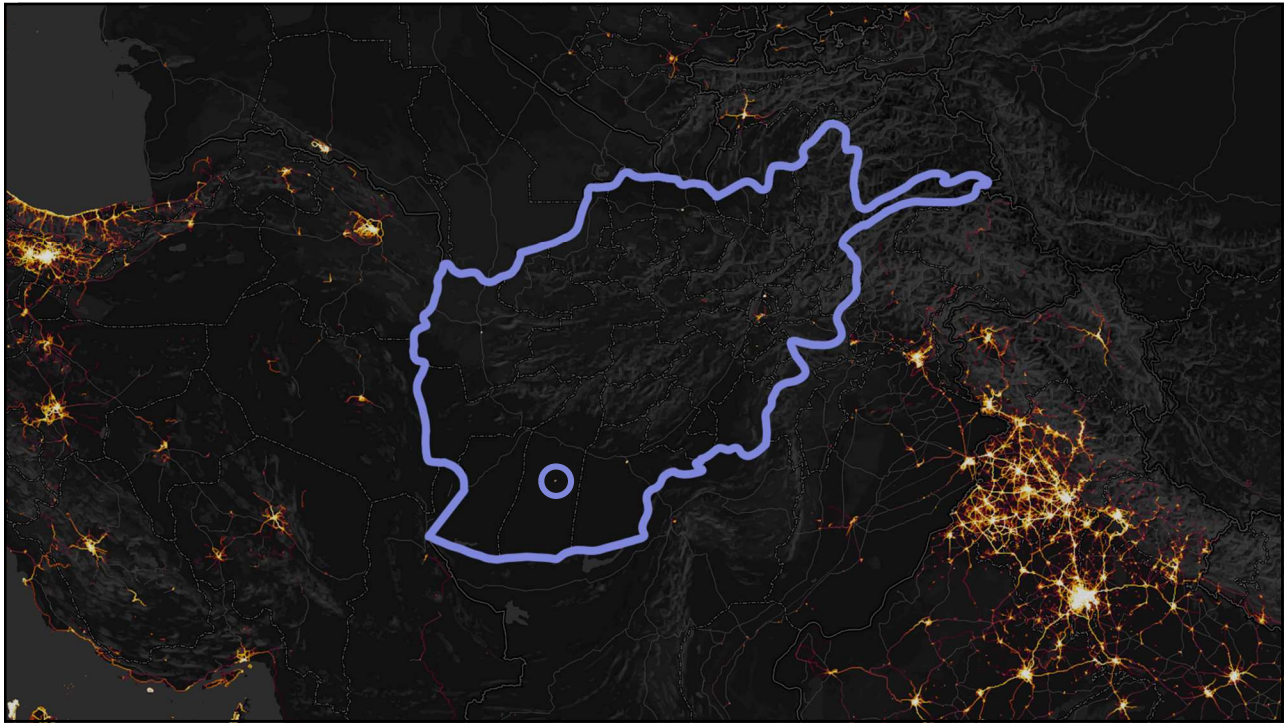
A Formal Introduction to Information Leakage

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July 17, 2021
The Diana Initiative

1

U.S. soldiers are revealing sensitive and dangerous information by jogging	The Strava Heat Map and the End of Secrets
<i>Strava Fitness App Can Reveal Military Sites, Analysts Say</i>	Fitness app Strava lights up staff at military bases

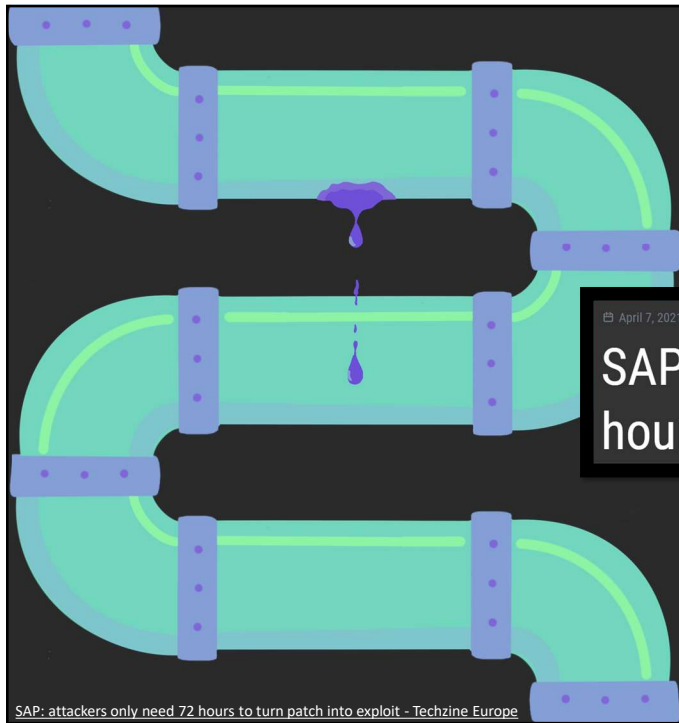
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Intentional Leakage?

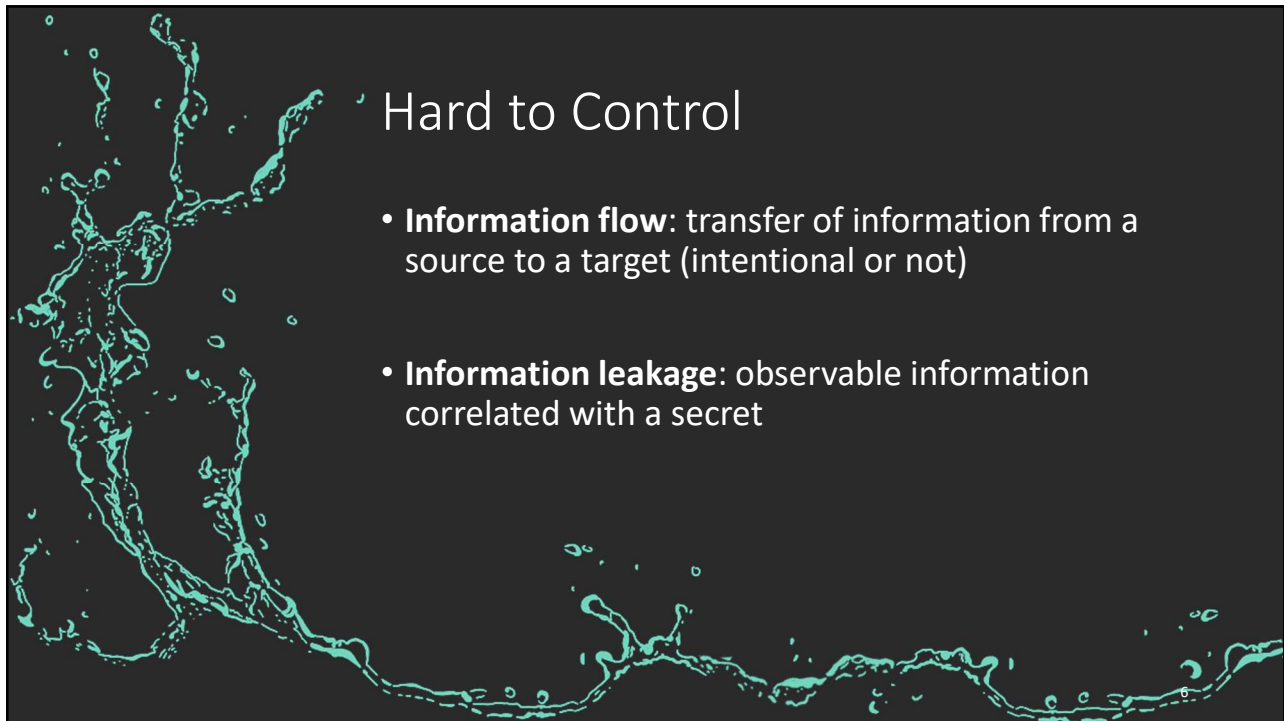
April 7, 2021 19:24 2 min Emile Witterman SECURITY

SAP: attackers only need 72 hours to turn patch into exploit

SAP: attackers only need 72 hours to turn patch into exploit - Techzine Europe

5

5



Hard to Control

- **Information flow:** transfer of information from a source to a target (intentional or not)
- **Information leakage:** observable information correlated with a secret

6

6

The Point

- You have a framework for information leakage
- You have the tools to calculate leakage
- You can apply this framework to compare systems & to reduce your attack surface

7

7

A (very) Brief Academic Timeline

Does it
flow?

1973 Bell & LaPadula: Write Up, Read Down

1982 Goguen & Meseguer: Noninterference: public output shouldn't tell you anything about secret input

How much
does it
flow?

1991 Grey: Use Mutual Information, Shannon entropy

What's the
value of the
flow?

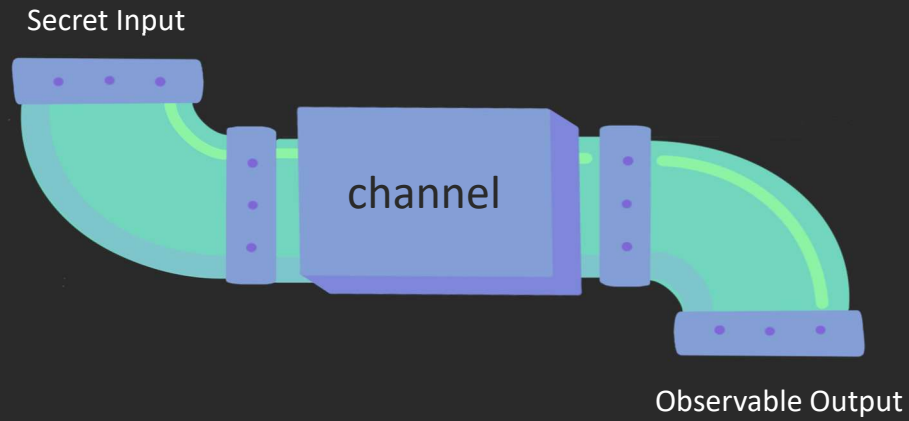
2009 Smith: Use min-entropy

2020 Alvim, Chatzikokolakis, McIver, Morgan, Palamidessi, & Smith: The Science of Quantitative Information Flow

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Quantitative Information Flow (QIF)



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Model the Adversary

Goal: Exact or approximate?

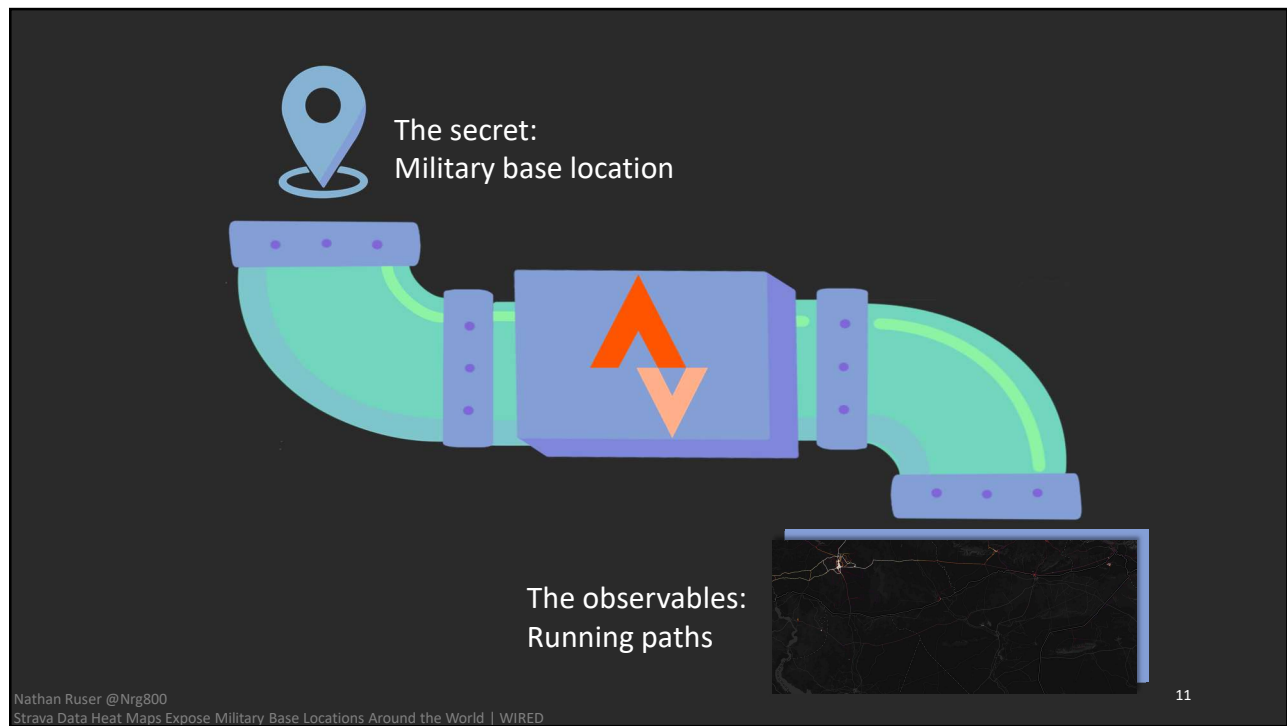
Abilities: Multiple chances? Constraints?

Cost: Consequences & rewards

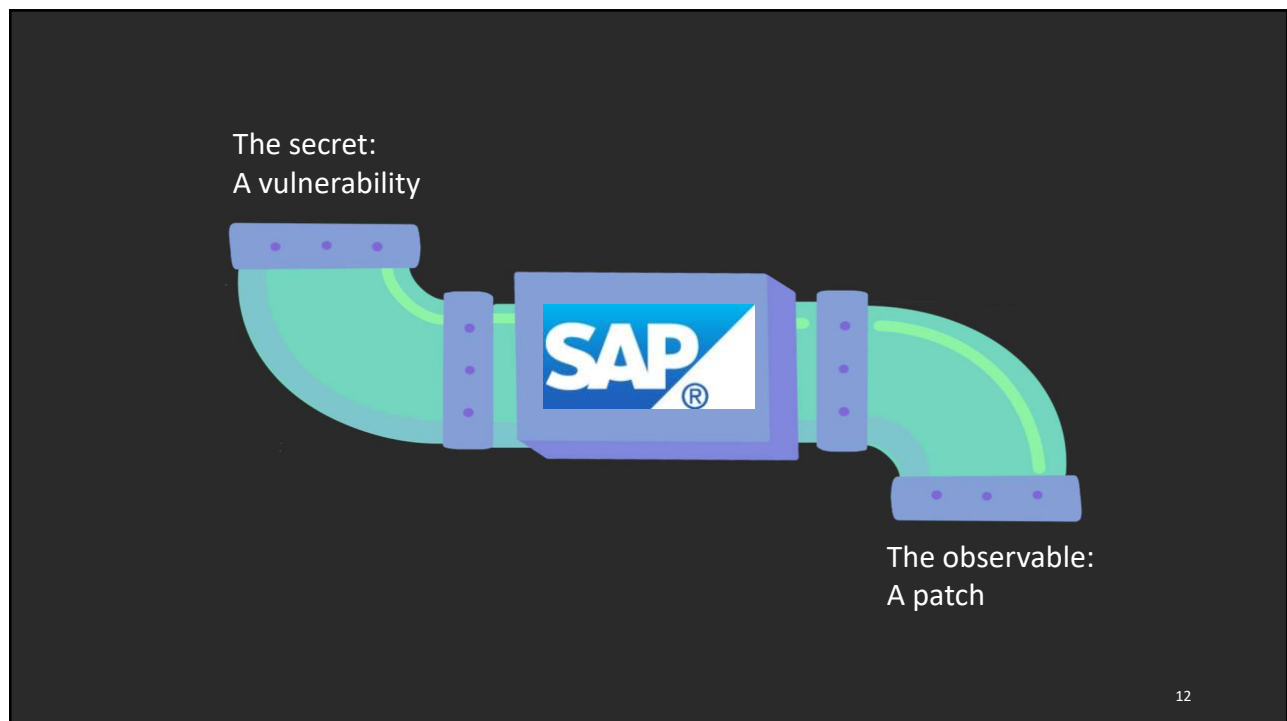


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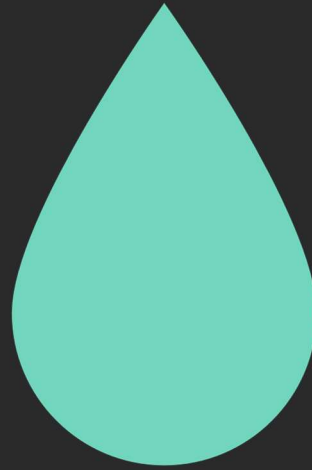
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12

Measuring Leakage

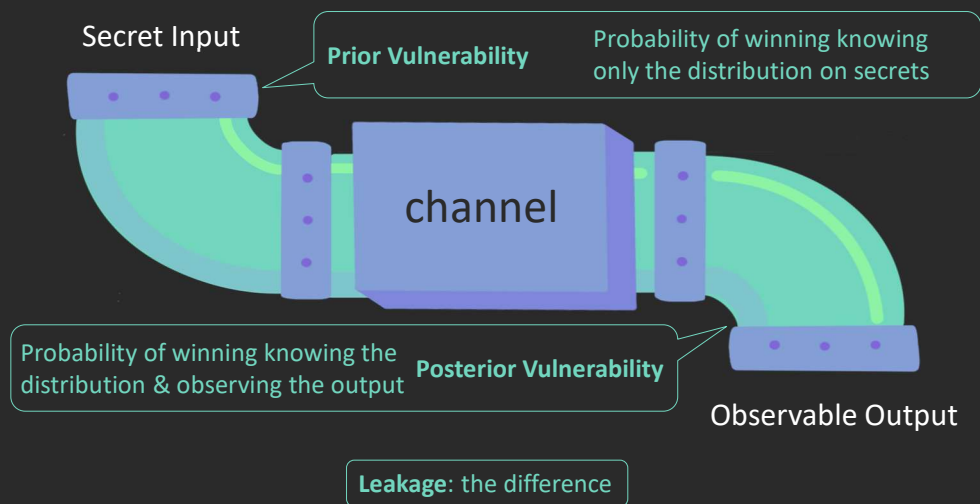
How to isolate the effect of a system



13

13

How to Measure Leakage



14

14

Calculating Leakage: Button vs Touchscreen



15

15



Setting Up

- Secret: 4-digit code
- Adversary: Bayes
 - Goal: guess entire secret exactly
 - Abilities: 1 try
 - Cost: All or nothing!

16

16

The Bayes Leakage Algorithm

1. Find distribution on secrets
2. Pick most likely secret $\leftarrow$ prior vulnerability
3. For all outputs:
 1. Isolate secrets that could produce the output
 2. Sum secrets' probabilities $\leftarrow$ prob. of output (PO)
 3. Normalize probabilities
 4. Pick most likely secret $\leftarrow$ prob. of success (PS)
4. $\Sigma(\text{PO} \times \text{PS})$ for all outputs $\leftarrow$ posterior vulnerability
5. Leakage: prior vs. posterior (+/x)

17

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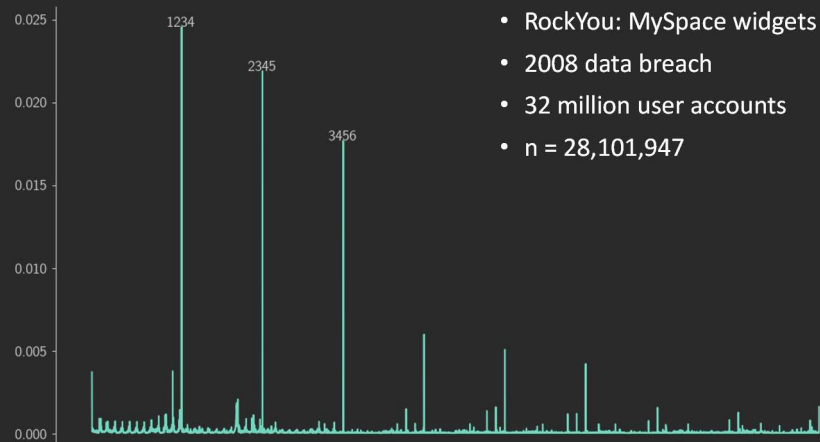
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18

18

1. Find Distribution on Secrets



<https://github.com/sr-lab/pin-bank/blob/master/frequency/descending/4digit.txt>

19

19

Bayes Leakage Algorithm

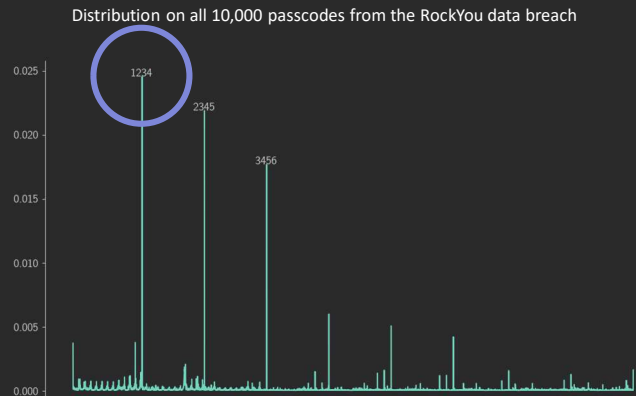
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20

20

2. Prior Vulnerability

- Adversary will choose secret with the highest probability
- Pick the most likely code, 1234
- Prior Bayes vulnerability = 0.02455



<https://github.com/sr-lab/pin-bank/blob/master/frequency/descending/4digit.txt>

21

21

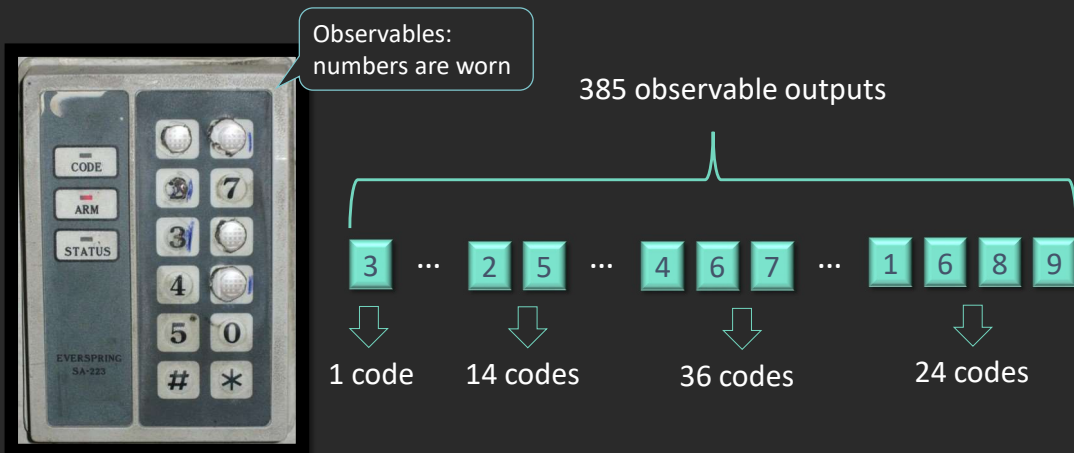
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4. $\sum(\text{PO} \times \text{PS})$ for all outputs $\leftarrow$ posterior vulnerability
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22

22

Observable Outputs

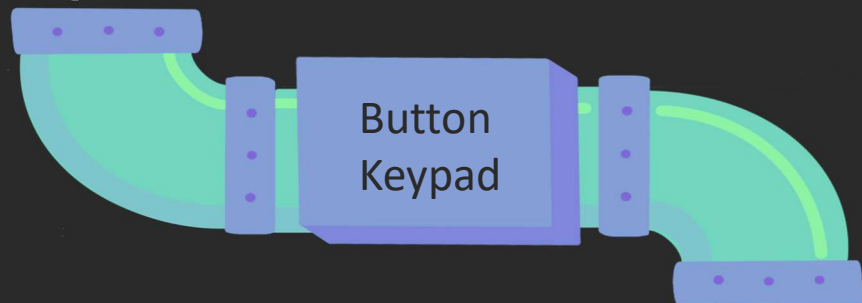


https://www.schneier.com/blog/archives/2009/07/information_lea_1.html

23

23

The secret:
4-digit code



The observables:
Worn numbers

24

24

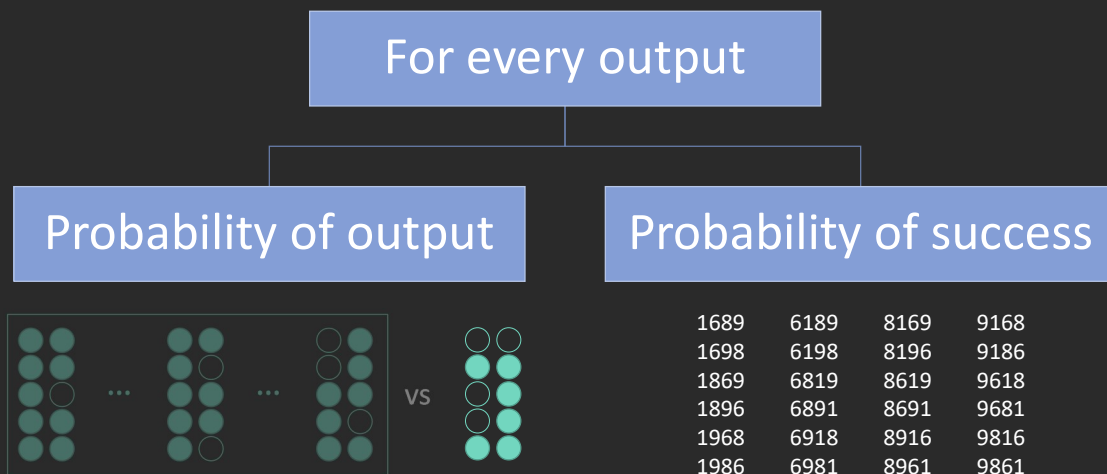
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25

25

Observable Outputs



26

26

3.1: Isolate Secrets

1689	6189	8169	9168
1698	6198	8196	9186
1869	6819	8619	9618
1896	6891	8691	9681
1968	6918	8916	9816
1986	6981	8961	9861

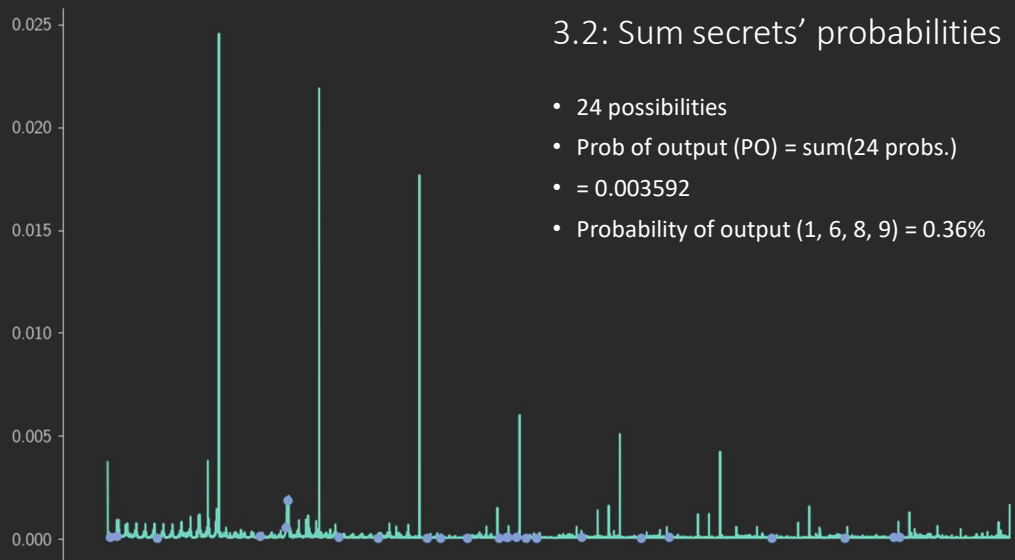


1 6 8 9

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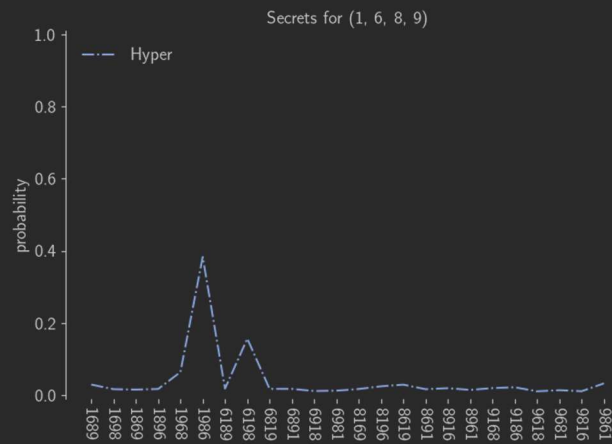
3.2: Sum secrets' probabilities



28

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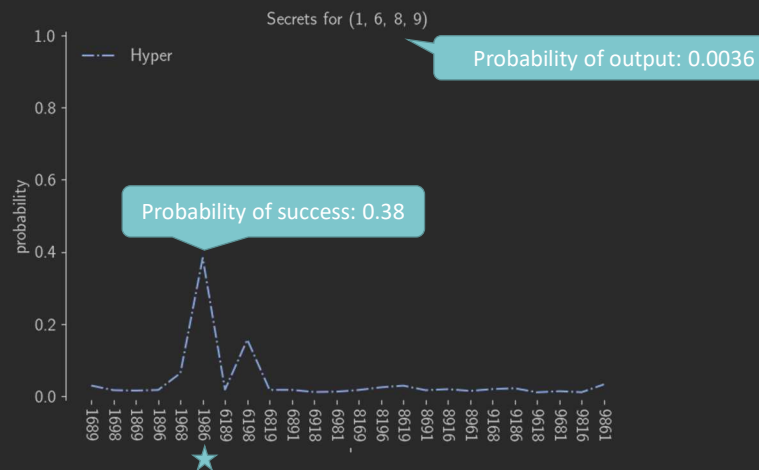
3.3: Normalize



29

29

3.4: Pick the Most Likely Secret



30

30

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31

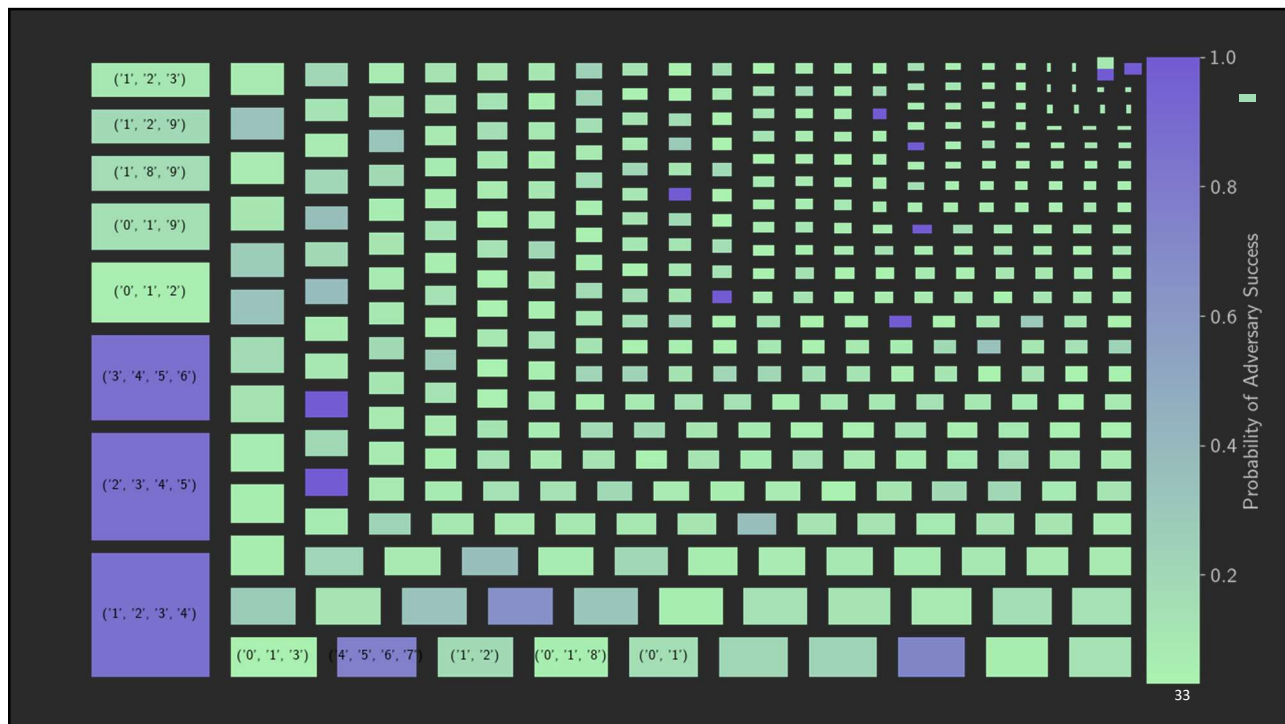
31

Posterior Vulnerability

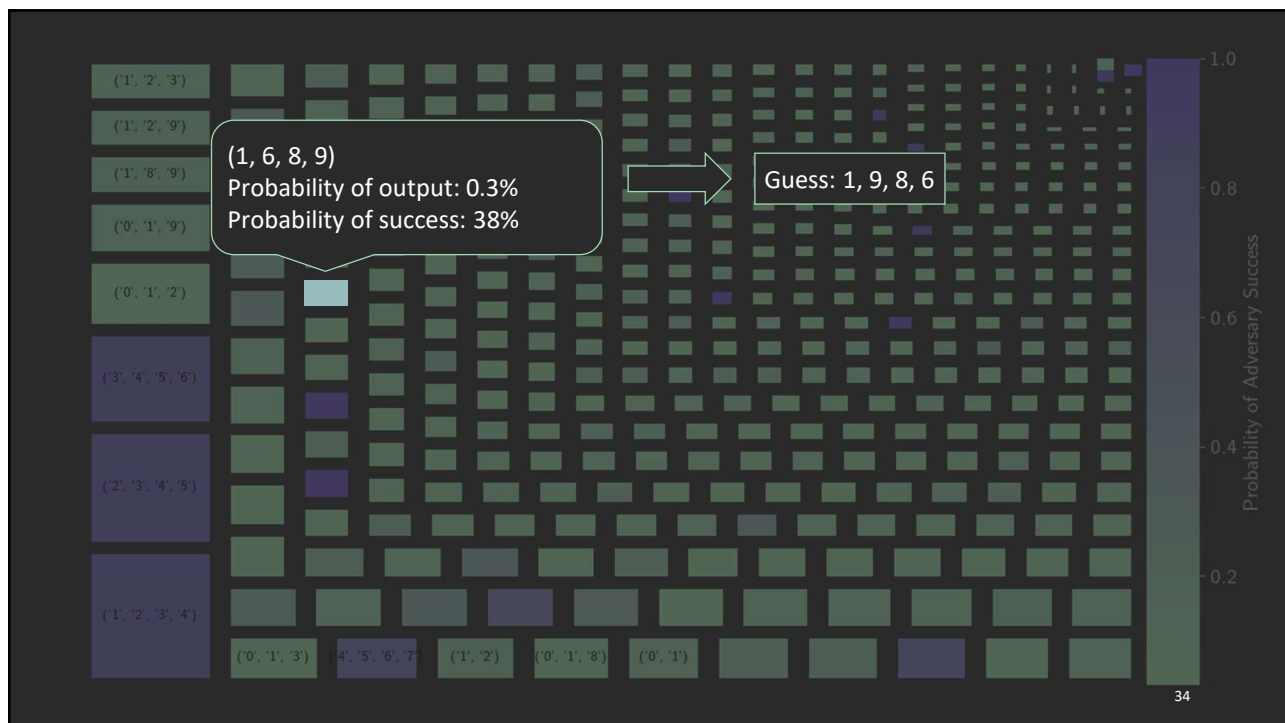
[0, '0']: 0.0037 x 1.0	[6, '7']: 0.0008 x 0.1236	[1, '3, '4']: 0.0034 x 0.1378	[3, '5, '8']: 0.0016 x 0.0368	[0, '1, '5, '9']: 0.0035 x 0.2109	[0, '4, '6, '9']: 0.0013 x 0.0948	[1, '3, '8, '9']: 0.0029 x 0.3199	[2, '5, '7, '8']: 0.0018 x 0.1466
[1, '1, '0']: 0.0036 x 1.0	[6, '8']: 0.0009 x 0.1258	[1, '3, '5']: 0.0026 x 0.0435	[3, '5, '9']: 0.0017 x 0.1041	[0, '1, '6, '7']: 0.0024 x 0.0919	[0, '4, '7, '8']: 0.0017 x 0.1193	[1, '4, '5, '9']: 0.0019 x 0.0506	[2, '5, '7, '9']: 0.0012 x 0.0627
[2, '1, '0011 x 1.0	[6, '9']: 0.0017 x 0.1303	[1, '3, '6']: 0.0023 x 0.0465	[3, '6, '7]: 0.0018 x 0.0447	[0, '1, '6, '8]: 0.0034 x 0.0881	[0, '4, '7, '9]: 0.0012 x 0.0909	[1, '4, '5, '9]: 0.0013 x 0.0811	[2, '5, '8, '9]: 0.0017 x 0.1023
[3, '1, '0007 x 1.0	[7, '8]: 0.0011 x 0.1365	[1, '3, '7]: 0.0022 x 0.0476	[3, '6, '8]: 0.0015 x 0.0376	[0, '1, '6, '9]: 0.0034 x 0.2123	[0, '4, '8, '9]: 0.002 x 0.1395	[1, '4, '5, '9]: 0.0016 x 0.1079	[2, '6, '7, '8]: 0.0012 x 0.0686
[4, '1, '0006 x 1.0	[7, '9]: 0.0008 x 0.107	[1, '3, '8]: 0.0027 x 0.0627	[3, '6, '9]: 0.0018 x 0.0401	[0, '1, '7, '8]: 0.003 x 0.0837	[0, '5, '6, '7]: 0.0011 x 0.067	[1, '4, '5, '9]: 0.0015 x 0.0648	[2, '6, '7, '9]: 0.0011 x 0.0569
[5, '1, '0016 x 1.0	[8, '9]: 0.0003 x 0.132	[1, '3, '9]: 0.0033 x 0.3284	[3, '7, '8]: 0.0015 x 0.0396	[0, '1, '7, '9]: 0.0036 x 0.1983	[0, '5, '6, '8]: 0.0019 x 0.1232	[1, '4, '5, '9]: 0.0011 x 0.0512	[2, '6, '8, '9]: 0.0014 x 0.073
[6, '1, '0012 x 1.0	[9, '1, '2]: 0.0133 x 0.0563	[1, '4, '5]: 0.0026 x 0.0769	[3, '7, '9]: 0.0013 x 0.0426	[0, '1, '8, '9]: 0.0059 x 0.14	[0, '5, '6, '9]: 0.0013 x 0.0827	[1, '4, '5, '9]: 0.0014 x 0.1376	[2, '7, '8, '9]: 0.0014 x 0.0783
[7, '1, '0016 x 1.0	[0, '1, '3]: 0.0085 x 0.0504	[1, '4, '6]: 0.0021 x 0.0619	[3, '8, '9]: 0.0015 x 0.0427	[0, '2, '3, '4]: 0.0023 x 0.0969	[0, '5, '7, '8]: 0.0019 x 0.13	[1, '4, '5, '9]: 0.0015 x 0.1168	[3, '4, '5, '6]: 0.0204 x 0.8674
[8, '1, '0013 x 1.0	[0, '1, '4]: 0.0063 x 0.0612	[1, '4, '7]: 0.0023 x 0.0481	[4, '5, '6]: 0.0024 x 0.0506	[0, '2, '3, '5]: 0.003 x 0.1138	[0, '5, '7, '9]: 0.0013 x 0.0902	[1, '4, '7, '8]: 0.0019 x 0.2621	[3, '4, '5, '7]: 0.0011 x 0.0686
[9, '1, '0016 x 1.0	[0, '1, '5]: 0.0066 x 0.0632	[1, '4, '8]: 0.0026 x 0.0573	[4, '5, '7]: 0.0016 x 0.034	[0, '2, '3, '6]: 0.0021 x 0.1034	[0, '5, '8, '9]: 0.0022 x 0.1104	[1, '4, '7, '9]: 0.0017 x 0.0482	[3, '4, '5, '8]: 0.0011 x 0.055
[0, '1, '0071 x 0.1581	[0, '1, '6]: 0.0058 x 0.0679	[1, '4, '9]: 0.005 x 0.3356	[4, '5, '8]: 0.0018 x 0.0499	[0, '2, '3, '7]: 0.0024 x 0.1015	[0, '6, '7, '8]: 0.0019 x 0.1471	[1, '4, '8, '9]: 0.0031 x 0.3575	[3, '4, '5, '9]: 0.001 x 0.0599
[0, '2, '0054 x 0.1809	[0, '2, '3]: 0.0058 x 0.0662	[1, '5, '6]: 0.0022 x 0.0783	[4, '5, '9]: 0.0015 x 0.0353	[0, '2, '3, '8]: 0.0026 x 0.0963	[0, '6, '7, '9]: 0.0012 x 0.0732	[1, '5, '6, '7]: 0.0011 x 0.0575	[3, '4, '6, '7]: 0.0011 x 0.1168
[0, '3, '0024 x 0.1759	[0, '2, '4]: 0.0076 x 0.0613	[1, '5, '7]: 0.0022 x 0.0531	[4, '6, '7]: 0.0014 x 0.0367	[0, '2, '3, '9]: 0.0026 x 0.0923	[0, '6, '8, '9]: 0.0021 x 0.1151	[1, '5, '6, '8]: 0.0013 x 0.0647	[3, '4, '6, '8]: 0.001 x 0.0547
[0, '4, '0017 x 0.2231	[0, '2, '5]: 0.0071 x 0.2055	[1, '5, '8]: 0.0026 x 0.0616	[4, '6, '8]: 0.0016 x 0.0383	[0, '2, '4, '5]: 0.0024 x 0.0784	[0, '7, '8, '9]: 0.0029 x 0.1847	[1, '5, '6, '9]: 0.0013 x 0.162	[3, '4, '6, '9]: 0.0009 x 0.0579
[0, '5, '002 x 0.2187	[0, '2, '6]: 0.0066 x 0.1336	[1, '5, '9]: 0.0052 x 0.2753	[4, '6, '9]: 0.0014 x 0.0348	[0, '2, '4, '6]: 0.002 x 0.0977	[1, '2, '3, '4]: 0.0287 x 0.8569	[1, '5, '7, '8]: 0.0013 x 0.0723	[3, '4, '7, '8]: 0.001 x 0.0648
[0, '6, '0017 x 0.2264	[0, '2, '7]: 0.006 x 0.1701	[1, '6, '7]: 0.002 x 0.0643	[4, '7, '8]: 0.0016 x 0.0386	[0, '2, '4, '7]: 0.0019 x 0.092	[1, '2, '3, '5]: 0.0031 x 0.1005	[1, '5, '7, '9]: 0.0023 x 0.1822	[3, '4, '7, '9]: 0.0008 x 0.0518
[0, '7, '0019 x 0.2333	[0, '2, '8]: 0.0071 x 0.2275	[1, '6, '8]: 0.0025 x 0.0606	[4, '7, '9]: 0.0013 x 0.0323	[0, '2, '4, '8]: 0.0024 x 0.0784	[1, '2, '3, '6]: 0.0039 x 0.1847	[1, '5, '8, '9]: 0.0013 x 0.162	[3, '4, '8, '9]: 0.0009 x 0.0579
[0, '8, '0023 x 0.1851	[0, '2, '9]: 0.0063 x 0.3141	[1, '6, '9]: 0.0048 x 0.2186	[4, '8, '9]: 0.0016 x 0.0353	[0, '2, '4, '9]: 0.0023 x 0.0787	[1, '2, '3, '7]: 0.002 x 0.1272	[1, '6, '7, '8]: 0.0013 x 0.1872	[3, '4, '9, '8]: 0.0011 x 0.0823
[0, '9, '0026 x 0.1848	[0, '3, '1]: 0.0064 x 0.3237	[1, '7, '8]: 0.0027 x 0.0758	[5, '6, '7]: 0.0016 x 0.0458	[0, '2, '5, '6]: 0.0022 x 0.1418	[1, '2, '3, '8]: 0.0023 x 0.0754	[1, '6, '7, '9]: 0.0019 x 0.2251	[3, '5, '8, '9]: 0.0014 x 0.1141
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[1, '3, '0019 x 0.1679	[0, '3, '4]: 0.003 x 0.1182	[2, '3, '4]: 0.0029 x 0.0715	[5, '7, '8]: 0.0017 x 0.0388	[0, '2, '5, '9]: 0.0025 x 0.1332	[1, '2, '4, '6]: 0.0017 x 0.056	[2, '3, '4, '5]: 0.0253 x 0.8668	[3, '5, '9, '9]: 0.0016 x 0.2018
[1, '4, '0019 x 0.163	[0, '3, '5]: 0.0031 x 0.1027	[2, '3, '5]: 0.0041 x 0.0789	[5, '7, '9]: 0.0017 x 0.1348	[0, '2, '6, '7]: 0.0017 x 0.0916	[1, '2, '4, '7]: 0.0021 x 0.1314	[2, '3, '4, '6]: 0.0015 x 0.0552	[3, '5, '9, '9]: 0.001 x 0.0552
[1, '5, '0016 x 0.1361	[0, '3, '6]: 0.0029 x 0.1099	[2, '3, '6]: 0.0023 x 0.0494	[5, '8, '9]: 0.0018 x 0.0408	[0, '2, '6, '8]: 0.0024 x 0.0801	[1, '2, '4, '8]: 0.0021 x 0.0766	[2, '3, '4, '7]: 0.0012 x 0.0598	[3, '5, '9, '9]: 0.0009 x 0.0563
[1, '6, '0017 x 0.1369	[0, '3, '7]: 0.0026 x 0.1086	[2, '3, '7]: 0.0019 x 0.0494	[6, '7, '8]: 0.0017 x 0.0454	[0, '2, '6, '9]: 0.0023 x 0.0992	[1, '2, '4, '9]: 0.0022 x 0.1239	[2, '3, '4, '8]: 0.0013 x 0.0663	[3, '5, '9, '9]: 0.0009 x 0.0631
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[1, '8, '0023 x 0.1322	[0, '3, '9]: 0.0034 x 0.0758	[2, '3, '9]: 0.0022 x 0.0442	[6, '8, '9]: 0.0017 x 0.0357	[0, '2, '7, '9]: 0.0023 x 0.1135	[1, '2, '5, '7]: 0.0017 x 0.0734	[2, '3, '5, '6]: 0.0025 x 0.2422	[3, '5, '9, '9]: 0.0011 x 0.1484
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[2, '2, '0026 x 0.1695	[0, '4, '3]: 0.0024 x 0.1198	[2, '4, '3]: 0.002 x 0.0485	[7, '1, '2, '4]: 0.0047 x 0.0841	[0, '3, '4, '5]: 0.0011 x 0.053	[1, '2, '6, '7]: 0.0014 x 0.0635	[2, '3, '5, '9]: 0.0014 x 0.1143	[4, '5, '9, '9]: 0.0016 x 0.2911
[2, '3, '0012 x 0.1746	[0, '4, '4]: 0.0026 x 0.0867	[2, '4, '4]: 0.0023 x 0.0581	[7, '1, '2, '5]: 0.0024 x 0.1116	[0, '3, '4, '6]: 0.0011 x 0.062	[1, '2, '6, '8]: 0.002 x 0.0943	[2, '3, '5, '9]: 0.0011 x 0.0557	[4, '5, '9, '9]: 0.0017 x 0.1404
[2, '4, '0012 x 0.1746	[0, '4, '5]: 0.0029 x 0.0815	[2, '4, '5]: 0.0021 x 0.0576	[7, '1, '2, '6]: 0.0041 x 0.1071	[0, '3, '4, '7]: 0.0011 x 0.062	[1, '2, '6, '9]: 0.0022 x 0.1107	[2, '3, '5, '9]: 0.0012 x 0.0715	[4, '5, '9, '9]: 0.0009 x 0.0538
[2, '5, '0014 x 0.141	[0, '5, '1]: 0.0027 x 0.1379	[2, '5, '1]: 0.0027 x 0.1379	[7, '1, '2, '7]: 0.0039 x 0.0987	[0, '3, '4, '8]: 0.0014 x 0.062	[1, '2, '7, '8]: 0.0021 x 0.1039	[2, '3, '5, '9]: 0.0018 x 0.1652	[4, '5, '9, '9]: 0.0016 x 0.299
[2, '6, '0013 x 0.1174	[0, '5, '2]: 0.0024 x 0.119	[2, '5, '2]: 0.0023 x 0.1127	[7, '1, '2, '8]: 0.002 x 0.1207	[0, '3, '4, '9]: 0.0013 x 0.0692	[1, '2, '7, '9]: 0.0025 x 0.1282	[2, '3, '5, '9]: 0.0017 x 0.137	[4, '5, '9, '9]: 0.001 x 0.0574
[2, '7, '0013 x 0.1694	[0, '5, '3]: 0.0018 x 0.0784	[2, '5, '3]: 0.002 x 0.1108	[7, '1, '2, '9]: 0.002 x 0.1207	[0, '3, '5, '6]: 0.0013 x 0.0692	[1, '2, '8, '8]: 0.0025 x 0.0966	[2, '3, '5, '9]: 0.0011 x 0.0696	[4, '5, '9, '9]: 0.001 x 0.1126
[2, '8, '0009 x 0.1059	[0, '5, '4]: 0.0031 x 0.095	[2, '5, '4]: 0.0026 x 0.1361	[7, '1, '3, '4]: 0.0025 x 0.0803	[0, '3, '5, '7]: 0.0018 x 0.0687	[1, '2, '8, '9]: 0.0017 x 0.0941	[2, '3, '5, '9]: 0.0015 x 0.1111	[4, '5, '9, '9]: 0.001 x 0.0628
[2, '9, '0011 x 0.1229	[0, '6, '1]: 0.0023 x 0.1291	[2, '6, '1]: 0.0018 x 0.0575	[7, '1, '3, '5]: 0.0017 x 0.081	[0, '3, '5, '8]: 0.0015 x 0.0919	[1, '3, '4, '6]: 0.0015 x 0.0925	[2, '4, '5, '6]: 0.0018 x 0.154	[4, '5, '9, '9]: 0.0019 x 0.2476
[3, '1, '0007 x 0.0943	[0, '6, '2]: 0.0026 x 0.1015	[2, '6, '2]: 0.0022 x 0.0486	[7, '1, '3, '6]: 0.0024 x 0.0991	[0, '3, '5, '9]: 0.0017 x 0.0558	[1, '3, '4, '7]: 0.0013 x 0.0728	[2, '4, '5, '7]: 0.0016 x 0.163	[4, '5, '9, '9]: 0.0017 x 0.2282
[3, '2, '0008 x 0.1141	[0, '6, '3]: 0.0031 x 0.1163	[2, '6, '3]: 0.002 x 0.0512	[7, '1, '3, '7]: 0.0024 x 0.0851	[0, '3, '6, '8]: 0.0016 x 0.1225	[1, '3, '4, '8]: 0.0014 x 0.0594	[2, '4, '5, '8]: 0.0016 x 0.0994	[4, '5, '9, '9]: 0.001 x 0.0711
[3, '3, '0008 x 0.0965	[0, '7, '1]: 0.0038 x 0.085	[2, '7, '1]: 0.0022 x 0.0565	[7, '1, '3, '8]: 0.0032 x 0.1031	[0, '3, '6, '9]: 0.0014 x 0.096	[1, '3, '4, '9]: 0.0013 x 0.0572	[2, '4, '5, '9]: 0.0012 x 0.0621	[4, '5, '9, '9]: 0.0015 x 0.0721
[4, '1, '0013 x 0.1514	[0, '7, '2]: 0.0029 x 0.0984	[2, '7, '2]: 0.0019 x 0.0549	[7, '1, '3, '9]: 0.0037 x 0.2018	[0, '3, '7, '8]: 0.0018 x 0.1198	[1, '3, '5, '6]: 0.0013 x 0.0519	[2, '4, '6, '7]: 0.0011 x 0.0583	[4, '5, '9, '9]: 0.0013 x 0.0902
[4, '2, '0008 x 0.117	[0, '7, '3]: 0.0026 x 0.1077	[2, '7, '3]: 0.0025 x 0.0584	[7, '1, '4, '5]: 0.0024 x 0.0923	[0, '3, '7, '9]: 0.0013 x 0.0954	[1, '3, '5, '7]: 0.0016 x 0.1729	[2, '4, '6, '8]: 0.0022 x 0.1115	[4, '5, '9, '9]: 0.0004 x 0.6603
[4, '3, '0008 x 0.1188	[0, '7, '4]: 0.0029 x 0.1063	[2, '7, '4]: 0.0021 x 0.0643	[7, '1, '4, '6]: 0.0022 x 0.0949	[0, '3, '8, '9]: 0.0014 x 0.096	[1, '3, '5, '8]: 0.0015 x 0.0651	[2, '4, '6, '9]: 0.0012 x 0.0631	
[4, '4, '0008 x 0.1171	[0, '7, '5]: 0.0027 x 0.1059	[2, '7, '5]: 0.002 x 0.0575	[7, '1, '4, '7]: 0.0024 x 0.0847	[0, '4, '5, '6]: 0.0013 x 0.0779	[1, '3, '5, '9]: 0.0018 x 0.1347	[2, '4, '7, '8]: 0.0012 x 0.0779	
[4, '5, '0007 x 0.1098	[0, '7, '6]: 0.0027 x 0.1059	[2, '7, '6]: 0.002 x 0.0575	[7, '1, '4, '8]: 0.0024 x 0.0847	[0, '4, '5, '7]: 0.0013 x 0.0779	[1, '3, '6, '6]: 0.0013 x 0.0562	[2, '4, '7, '9]: 0.0011 x 0.0603	
[4, '6, '0013 x 0.1833	[0, '7, '7]: 0.0027 x 0.1059	[2, '7, '7]: 0.002 x 0.0575	[7, '1, '4, '9]: 0.0024 x 0.0847	[0, '4, '5, '8]: 0.0013 x 0.0779	[1, '3, '6, '7]: 0.0013 x 0.0562	[2, '4, '8, '9]: 0.0011 x 0.0603	
[4, '7, '0008 x 0.106	[0, '7, '8]: 0.0027 x 0.1059	[2, '7, '8]: 0.002 x 0.0575	[7, '1, '5, '1]: 0.0024 x 0.0847	[0, '4, '5, '9]: 0.0013 x 0.0779	[1, '3, '6, '8]: 0.0013 x 0.0562	[2, '4, '9, '9]: 0.0011 x 0.0603	
[4, '8, '0009 x 0.1273	[0, '7, '9]: 0.0027 x 0.1059	[2, '7, '9]: 0.002 x 0.0575	[7, '1, '5, '2]: 0.0024 x 0.0847	[0, '4, '6, '7]: 0.0013 x 0.0779	[1, '3, '6, '9]: 0.0013 x 0.0562	[2, '5, '6, '7]: 0.0011 x 0.0636	
[4, '9, '0008 x 0.1088	[0, '7, '10]: 0.0027 x 0.1059	[2, '7, '10]: 0.002 x 0.0575	[7, '1, '5, '3]: 0.0024 x 0.0847	[0, '4, '6, '8]: 0.0013 x 0.0779	[1, '3, '7, '8]: 0.0013 x 0.0686	[2, '5, '6, '8]: 0.0015 x 0.0714	
					[1, '3, '7, '9]: 0.0012 x 0.1824	[2, '5, '6, '9]: 0.0014 x 0.0985	

32

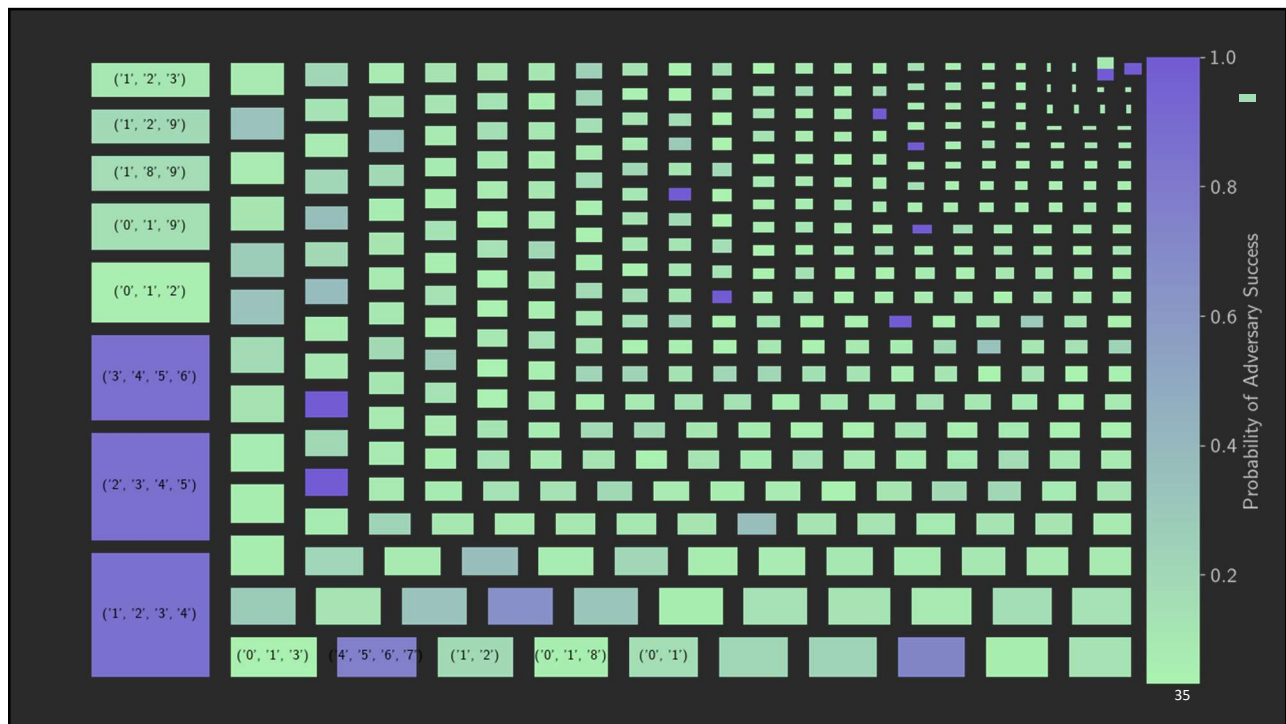
32



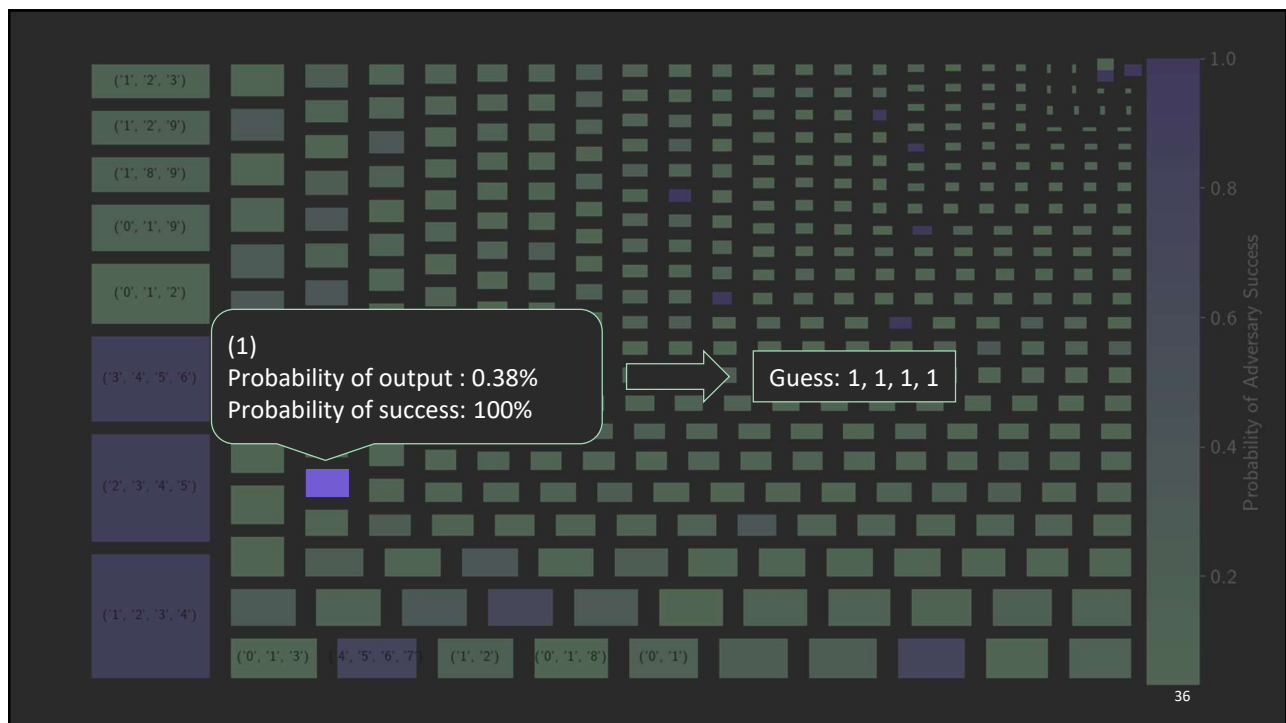
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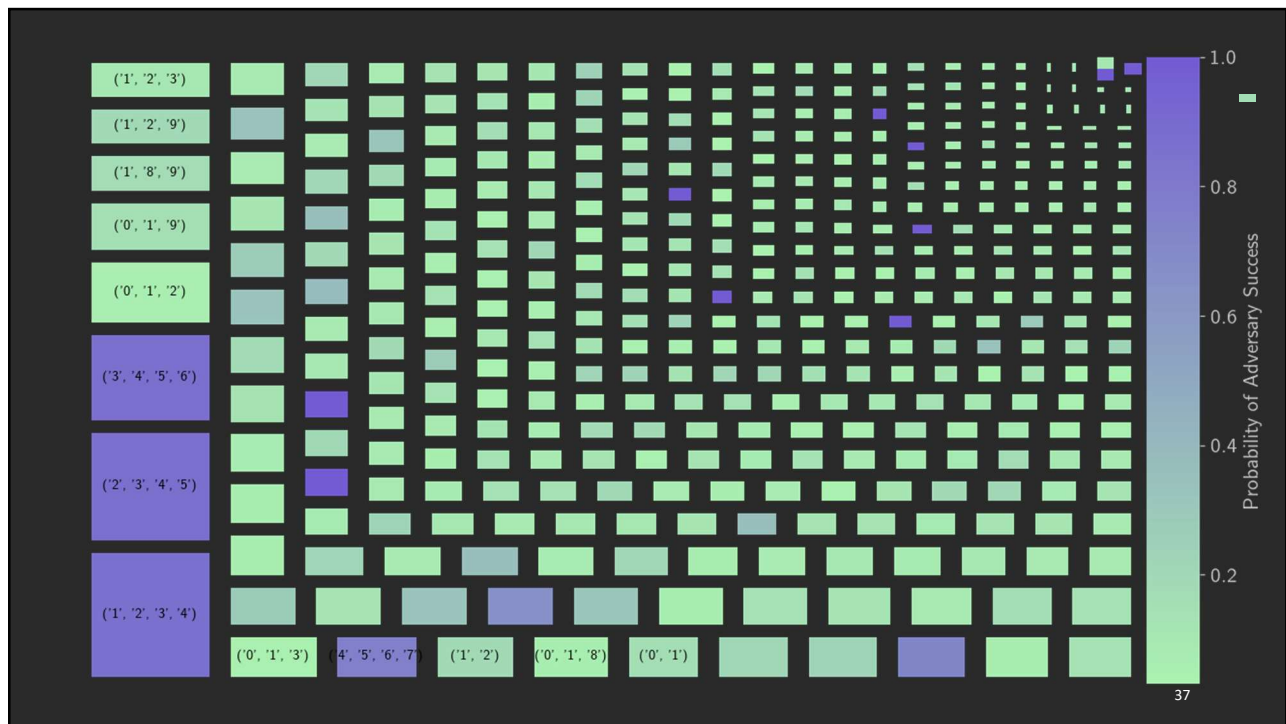
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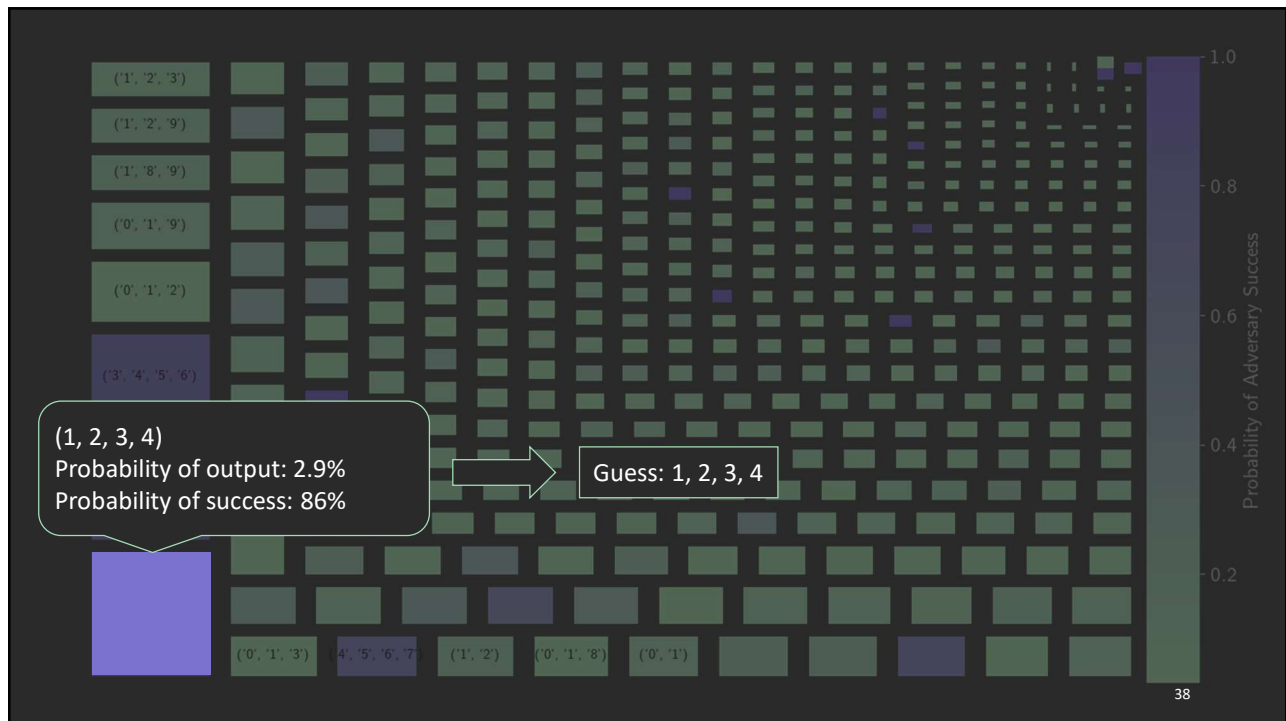
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36



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38

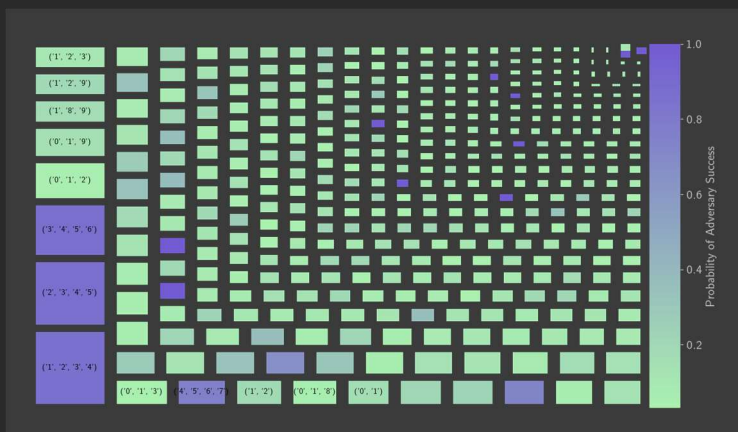
Bayes Leakage Algorithm

1. Find distribution on secrets
2. Pick most likely secret $\leftarrow$ prior vulnerability
3. For all outputs:
 1. Isolate secrets that could produce the output
 2. Sum secrets' probabilities $\leftarrow$ prob. of output (PO)
 3. Normalize probabilities
 4. Pick most likely secret $\leftarrow$ prob. of success (PS)
4. $\Sigma(\text{PO} \times \text{PS})$ for all outputs $\leftarrow$ posterior vulnerability
5. Leakage: prior vs. posterior (+/x)

39

39

Posterior Bayes Vulnerability



- $\text{sum}(\text{prob. output} \times \text{prob. success})$
- Posterior Bayes vulnerability ≈ 0.2047

40

40

Posterior Vulnerability

(0', 0): 0.0037 x 1.0	(6', 7'): 0.0008 x 0.1236	(1', 3', 4'): 0.0034 x 0.1378	(3', 5', 8'): 0.0016 x 0.0368	(0', 1', 5', 9'): 0.0025 x 0.2109	(0', 4', 6', 9'): 0.0013 x 0.0948	(1', 3', 8', 9'): 0.0029 x 0.3199	(2', 5', 7', 8'): 0.0018 x 0.1466
(1', 1): 0.0038 x 1.0	(6', 8'): 0.0009 x 0.1258	(1', 3', 5'): 0.0026 x 0.0445	(3', 5', 9'): 0.0017 x 0.1041	(0', 1', 6', 7'): 0.002 x 0.0919	(0', 4', 7', 8'): 0.0017 x 0.1193	(1', 4', 5', 6'): 0.0019 x 0.2056	(2', 5', 7', 9'): 0.0012 x 0.0627
(2', 2): 0.0011 x 1.0	(6', 9'): 0.0017 x 0.3403	(1', 3', 6'): 0.0023 x 0.0465	(3', 6', 7'): 0.0014 x 0.0447	(0', 1', 6', 8'): 0.003 x 0.0881	(0', 4', 7', 9'): 0.0012 x 0.0909	(1', 4', 5', 7'): 0.0013 x 0.0811	(2', 5', 8', 9'): 0.0017 x 0.1023
(3', 3): 0.0007 x 1.0	(7', 8'): 0.0011 x 0.1365	(1', 3', 7'): 0.0022 x 0.0476	(3', 6', 8'): 0.0015 x 0.0376	(0', 1', 6', 9'): 0.0034 x 0.2123	(0', 4', 8', 9'): 0.002 x 0.1395	(1', 4', 5', 9'): 0.0016 x 0.1079	(2', 6', 7', 8'): 0.0012 x 0.0686
(4', 4): 0.0058 x 1.0	(7', 9'): 0.0008 x 0.1107	(1', 3', 8'): 0.0027 x 0.0627	(3', 6', 9'): 0.0018 x 0.0401	(0', 1', 7', 8'): 0.003 x 0.0837	(0', 5', 6', 7'): 0.0011 x 0.067	(1', 4', 5', 9'): 0.0015 x 0.0648	(2', 6', 7', 9'): 0.0011 x 0.0569
(5', 5): 0.0016 x 1.0	(8', 9'): 0.0013 x 0.132	(1', 3', 9'): 0.0053 x 0.3284	(3', 7', 8'): 0.0015 x 0.0396	(0', 1', 7', 9'): 0.0036 x 0.1983	(0', 5', 6', 8'): 0.0019 x 0.1232	(1', 4', 6', 9'): 0.0011 x 0.0512	(2', 6', 8', 9'): 0.0014 x 0.073
(6', 6): 0.0012 x 1.0	(0', 1', 2'): 0.0153 x 0.0563	(1', 4', 5'): 0.0026 x 0.0769	(3', 7', 9'): 0.0013 x 0.0426	(0', 1', 8', 9'): 0.0059 x 0.14	(0', 5', 6', 9'): 0.0013 x 0.0827	(1', 4', 6', 8'): 0.0014 x 0.1376	(2', 7', 8', 9'): 0.0014 x 0.0783
(7', 7): 0.0016 x 1.0	(0', 1', 3'): 0.0085 x 0.0504	(1', 4', 6'): 0.0021 x 0.0619	(3', 8', 9'): 0.0015 x 0.0427	(0', 2', 3', 4'): 0.0023 x 0.0969	(0', 5', 7', 8'): 0.0019 x 0.13	(1', 4', 6', 9'): 0.0015 x 0.1168	(3', 4', 5', 8'): 0.0024 x 0.8574
(8', 8): 0.0013 x 1.0	(0', 1', 4'): 0.0063 x 0.0612	(1', 4', 7'): 0.0023 x 0.0481	(4', 5', 6'): 0.0024 x 0.0506	(0', 2', 3', 5'): 0.003 x 0.1138	(0', 5', 7', 9'): 0.0013 x 0.0902	(1', 4', 7', 8'): 0.0019 x 0.2261	(3', 4', 5', 7'): 0.0011 x 0.0686
(9', 9): 0.0016 x 1.0	(0', 1', 5'): 0.0066 x 0.0632	(1', 4', 8'): 0.0026 x 0.0573	(4', 5', 7'): 0.0016 x 0.034	(0', 2', 3', 6'): 0.0021 x 0.1034	(0', 5', 8', 9'): 0.0022 x 0.1104	(1', 4', 7', 9'): 0.0017 x 0.2042	(3', 4', 5', 8'): 0.0011 x 0.055
(0', 1'): 0.0071 x 0.1581	(0', 1', 6'): 0.0058 x 0.0679	(1', 4', 9'): 0.005 x 0.3356	(4', 5', 8'): 0.0018 x 0.0489	(0', 2', 3', 7'): 0.002 x 0.1015	(0', 6', 7', 8'): 0.0019 x 0.1471	(1', 4', 8', 9'): 0.0031 x 0.2575	(3', 4', 6', 9'): 0.001 x 0.0599
(0', 2'): 0.0054 x 0.1889	(0', 1', 7'): 0.0058 x 0.0652	(1', 5', 5'): 0.0022 x 0.0783	(4', 5', 9'): 0.0015 x 0.0353	(0', 2', 3', 8'): 0.0026 x 0.0963	(0', 6', 7', 9'): 0.0012 x 0.0732	(1', 5', 6', 7'): 0.0011 x 0.0575	(3', 4', 6', 7'): 0.0011 x 0.1168
(0', 3'): 0.0024 x 0.1752	(0', 1', 8'): 0.0076 x 0.0613	(1', 5', 7'): 0.0022 x 0.0531	(4', 6', 7'): 0.0014 x 0.0367	(0', 2', 3', 9'): 0.0026 x 0.0823	(0', 6', 8', 9'): 0.0021 x 0.1151	(1', 5', 6', 8'): 0.0013 x 0.0647	(3', 4', 6', 8'): 0.001 x 0.0547
(0', 4'): 0.0017 x 0.2231	(0', 1', 9'): 0.0123 x 0.1473	(1', 5', 8'): 0.0026 x 0.0616	(4', 6', 8'): 0.0016 x 0.0383	(0', 2', 4', 5'): 0.0024 x 0.1349	(0', 7', 8', 9'): 0.0029 x 0.1839	(1', 5', 6', 9'): 0.0017 x 0.1143	(3', 4', 6', 9'): 0.0009 x 0.0508
(0', 5'): 0.002 x 0.2187	(0', 2', 3'): 0.0066 x 0.1336	(1', 5', 9'): 0.0022 x 0.2753	(4', 6', 9'): 0.0014 x 0.0348	(0', 2', 4', 6'): 0.002 x 0.0977	(1', 2', 3', 4'): 0.0087 x 0.8569	(1', 5', 7', 8'): 0.0013 x 0.0723	(3', 4', 7', 8'): 0.001 x 0.0648
(0', 6'): 0.0017 x 0.2264	(0', 2', 4'): 0.006 x 0.1701	(1', 6', 7'): 0.002 x 0.0643	(4', 7', 8'): 0.0016 x 0.0386	(0', 2', 4', 7'): 0.0019 x 0.092	(1', 2', 3', 5'): 0.0031 x 0.1005	(1', 5', 7', 9'): 0.0023 x 0.1822	(3', 4', 7', 8'): 0.0008 x 0.0518
(0', 7'): 0.0019 x 0.2133	(0', 2', 5'): 0.0071 x 0.2055	(1', 6', 8'): 0.0025 x 0.0696	(4', 7', 9'): 0.0013 x 0.0323	(0', 2', 4', 8'): 0.0024 x 0.0784	(1', 2', 3', 6'): 0.0029 x 0.1847	(1', 5', 8', 9'): 0.0035 x 0.362	(3', 4', 8', 9'): 0.0009 x 0.0579
(0', 8'): 0.0023 x 0.1851	(0', 2', 6'): 0.0063 x 0.3141	(1', 6', 9'): 0.0048 x 0.2186	(4', 8', 9'): 0.0016 x 0.0353	(0', 2', 4', 9'): 0.0023 x 0.0707	(1', 2', 3', 7'): 0.002 x 0.1272	(1', 6', 7', 8'): 0.0013 x 0.0677	(3', 5', 6', 7'): 0.0011 x 0.0823
(0', 9'): 0.0026 x 0.1848	(0', 2', 7'): 0.0064 x 0.2327	(1', 7', 8'): 0.0027 x 0.0758	(5', 6', 7'): 0.0016 x 0.0458	(0', 2', 5', 6'): 0.0022 x 0.1418	(1', 2', 3', 8'): 0.0023 x 0.0754	(1', 6', 7', 9'): 0.0019 x 0.2251	(3', 5', 6', 8'): 0.0014 x 0.1141
(1', 1'): 0.0076 x 0.1893	(0', 2', 8'): 0.0071 x 0.2275	(1', 7', 9'): 0.0055 x 0.127	(5', 6', 8'): 0.0018 x 0.0493	(0', 2', 5', 7'): 0.0021 x 0.1397	(1', 2', 3', 9'): 0.0026 x 0.1239	<u>(1', 6', 8', 9'): 0.0036 x 0.3831</u>	(3', 5', 6', 9'): 0.0011 x 0.0987
(1', 3'): 0.0026 x 0.2081	(0', 2', 9'): 0.0064 x 0.1155	(1', 8', 9'): 0.01 x 0.1752	(5', 6', 9'): 0.0016 x 0.0351	(0', 2', 5', 8'): 0.0031 x 0.1165	(1', 2', 4', 5'): 0.0028 x 0.0771	(1', 7', 8', 9'): 0.0047 x 0.3589	(3', 5', 7', 8'): 0.0011 x 0.0748
(1', 4'): 0.0019 x 0.1679	(0', 3', 4'): 0.003 x 0.1182	(2', 3', 4'): 0.0029 x 0.0715	(5', 7', 8'): 0.0017 x 0.0388	(0', 2', 5', 9'): 0.0025 x 0.1382	(1', 2', 4', 6'): 0.0017 x 0.056	(2', 3', 4', 5'): 0.0053 x 0.8668	(3', 5', 7', 9'): 0.0016 x 0.2018
(1', 5'): 0.0019 x 0.1683	(0', 3', 5'): 0.0031 x 0.1027	(2', 3', 5'): 0.0041 x 0.0789	(5', 7', 9'): 0.0017 x 0.1848	(0', 2', 6', 7'): 0.0017 x 0.0916	(1', 2', 4', 7'): 0.0021 x 0.1314	(2', 3', 4', 6'): 0.0015 x 0.0748	(3', 5', 8', 9'): 0.001 x 0.0552
(1', 6'): 0.0016 x 0.1361	(3', 6', 6'): 0.0029 x 0.1099	(2', 3', 6'): 0.0023 x 0.0494	(5', 8', 9'): 0.0018 x 0.0408	(0', 2', 6', 8'): 0.0024 x 0.0801	(1', 2', 4', 8'): 0.0021 x 0.0766	(2', 3', 4', 7'): 0.0012 x 0.0598	(3', 6', 7', 8'): 0.0009 x 0.0565
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(1', 8'): 0.0023 x 0.1322	(0', 3', 8'): 0.0038 x 0.0791	(2', 3', 8'): 0.0023 x 0.0481	(6', 7', 9'): 0.0014 x 0.0377	(0', 2', 7', 8'): 0.0024 x 0.0638	(1', 2', 5', 6'): 0.0023 x 0.1645	(2', 3', 4', 9'): 0.0012 x 0.0668	(3', 6', 8', 9'): 0.0015 x 0.1575
(1', 9'): 0.0064 x 0.2851	(2', 3', 9'): 0.0034 x 0.0758	(2', 3', 9'): 0.0022 x 0.0442	(6', 8', 9'): 0.0017 x 0.0357	(0', 2', 7', 9'): 0.0023 x 0.1115	(1', 2', 5', 7'): 0.0017 x 0.0734	(2', 3', 5', 6'): 0.0025 x 0.1242	(3', 7', 8', 9'): 0.0011 x 0.1484
(2', 2'): 0.0027 x 0.2009	(0', 4', 5'): 0.0029 x 0.1293	(2', 4', 5'): 0.0032 x 0.0826	(7', 8', 9'): 0.0022 x 0.0452	(0', 2', 8', 9'): 0.0031 x 0.1091	(1', 2', 5', 8'): 0.0025 x 0.0815	(2', 3', 5', 7'): 0.0015 x 0.1101	(4', 5', 6', 7'): 0.0079 x 0.7552
(2', 4'): 0.0016 x 0.1929	(0', 4', 6'): 0.0024 x 0.1351	(2', 4', 6'): 0.0022 x 0.0515	(0', 1', 2', 3'): 0.0063 x 0.1448	(0', 3', 4', 5'): 0.0014 x 0.061	(1', 2', 5', 9'): 0.0025 x 0.1376	(2', 3', 5', 9'): 0.002 x 0.1028	(4', 5', 6', 8'): 0.0014 x 0.1001
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(2', 6'): 0.0012 x 0.1746	(0', 4', 8'): 0.0036 x 0.0867	(2', 4', 8'): 0.0023 x 0.051	(0', 1', 2', 5'): 0.0052 x 0.1116	(0', 3', 4', 7'): 0.0011 x 0.062	(1', 2', 6', 8'): 0.002 x 0.0943	(2', 3', 6', 7'): 0.0011 x 0.0557	(4', 5', 7', 8'): 0.0017 x 0.1404
(2', 7'): 0.0011 x 0.1567	(0', 4', 9'): 0.0029 x 0.0815	(2', 4', 9'): 0.0021 x 0.0576	(0', 1', 2', 6'): 0.0041 x 0.1071	(0', 3', 4', 8'): 0.0017 x 0.1058	(1', 2', 6', 9'): 0.0022 x 0.1107	(2', 3', 6', 8'): 0.0012 x 0.0714	(4', 5', 7', 9'): 0.0009 x 0.0538
(2', 8'): 0.0014 x 0.141	(0', 4', 10'): 0.0027 x 0.1307	(2', 4', 10'): 0.0023 x 0.1399	(0', 1', 2', 7'): 0.0039 x 0.0987	(0', 3', 4', 9'): 0.0014 x 0.0674	(1', 2', 7', 8'): 0.0021 x 0.1039	(2', 3', 6', 9'): 0.0018 x 0.1652	(4', 5', 8', 9'): 0.0016 x 0.299
(2', 9'): 0.0013 x 0.1174	(0', 5', 5'): 0.0024 x 0.119	(2', 5', 5'): 0.0023 x 0.1327	(0', 1', 2', 8'): 0.0049 x 0.0848	(0', 3', 5', 6'): 0.0013 x 0.0692	(1', 2', 7', 9'): 0.0025 x 0.1282	(2', 3', 7', 8'): 0.0014 x 0.137	(4', 6', 7', 8'): 0.001 x 0.0574
(3', 4'): 0.0013 x 0.1694	(0', 5', 6'): 0.0038 x 0.0878	(2', 5', 6'): 0.003 x 0.1108	(0', 1', 2', 9'): 0.0062 x 0.1207	(0', 3', 5', 7'): 0.0012 x 0.0839	(1', 2', 8', 9'): 0.0045 x 0.2087	(2', 3', 7', 9'): 0.0011 x 0.0696	(4', 6', 7', 9'): 0.001 x 0.1126
(3', 5'): 0.0009 x 0.1059	(0', 5', 7'): 0.0031 x 0.095	(2', 5', 7'): 0.0026 x 0.1361	(0', 1', 2', 10'): 0.0025 x 0.0803	(0', 3', 5', 8'): 0.0018 x 0.1082	(1', 3', 4', 5'): 0.0017 x 0.0825	(2', 3', 8', 9'): 0.0015 x 0.0811	(4', 6', 8', 9'): 0.001 x 0.0628
(3', 6'): 0.001 x 0.1229	(0', 5', 8'): 0.0023 x 0.1191	(2', 5', 8'): 0.0018 x 0.0975	(0', 1', 3', 5'): 0.0027 x 0.0881	(0', 3', 5', 9'): 0.0015 x 0.0919	(1', 3', 4', 6'): 0.0015 x 0.0941	(2', 4', 5', 6'): 0.0018 x 0.1154	(4', 6', 9', 9'): 0.0019 x 0.2476
(3', 7'): 0.0007 x 0.0943	(0', 6', 6'): 0.0036 x 0.1014	(2', 6', 6'): 0.0022 x 0.0486	(0', 1', 3', 6'): 0.0024 x 0.0991	(0', 3', 6', 7'): 0.0011 x 0.0558	(1', 3', 4', 7'): 0.0013 x 0.0594	(2', 4', 5', 7'): 0.0016 x 0.163	(5', 6', 7', 8'): 0.007 x 0.7282
(3', 8'): 0.0008 x 0.1141	(0', 6', 7'): 0.0031 x 0.1163	(2', 6', 7'): 0.002 x 0.0512	(0', 1', 3', 7'): 0.0024 x 0.0851	(0', 3', 6', 8'): 0.0016 x 0.1225	(1', 3', 4', 8'): 0.0014 x 0.0594	(2', 4', 5', 8'): 0.0016 x 0.0994	(5', 6', 7', 9'): 0.001 x 0.0711
(3', 9'): 0.0008 x 0.0965	(0', 7', 7'): 0.0038 x 0.085	(2', 7', 7'): 0.0022 x 0.0565	(0', 1', 3', 8'): 0.0023 x 0.1031	(0', 3', 6', 9'): 0.0014 x 0.096	(1', 3', 4', 9'): 0.0013 x 0.0572	(2', 4', 5', 9'): 0.0012 x 0.0621	(5', 6', 8', 9'): 0.0015 x 0.0721
(4', 4'): 0.0003 x 0.1514	(0', 7', 8'): 0.0029 x 0.0884	(2', 7', 8'): 0.0019 x 0.0549	(0', 1', 3', 9'): 0.0037 x 0.2018	(0', 3', 7', 8'): 0.0018 x 0.1198	(1', 3', 5', 6'): 0.0013 x 0.0519	(2', 4', 6', 7'): 0.0011 x 0.0583	(5', 6', 9', 9'): 0.0013 x 0.0902
(4', 5'): 0.0018 x 0.1574	(0', 8', 9'): 0.0046 x 0.0707	(2', 8', 9'): 0.0025 x 0.0584	(0', 1', 4', 5'): 0.0024 x 0.0933	(0', 3', 7', 9'): 0.0013 x 0.0954	(1', 3', 5', 7'): 0.0016 x 0.1729	(2', 4', 6', 8'): 0.0022 x 0.2115	<u>(5', 7', 8', 9'): 0.0054 x 0.6603</u>
(4', 6'): 0.0008 x 0.1188	(1', 2', 3'): 0.0097 x 0.1063	(3', 4', 5'): 0.0021 x 0.0643	(0', 1', 4', 6'): 0.0022 x 0.0949	(0', 3', 8', 9'): 0.002 x 0.106	(1', 3', 5', 8'): 0.0015 x 0.0651	(2', 4', 6', 9'): 0.0012 x 0.0631	(5', 7', 9', 9'): 0.0012 x 0.0631
(4', 7'): 0.0008 x 0.1174	(1', 2', 4'): 0.0025 x 0.0752	(3', 4', 6'): 0.0016 x 0.037	(0', 1', 4', 7'): 0.0021 x 0.0947	(0', 4', 5', 6'): 0.0013 x 0.0729	(1', 3', 5', 9'): 0.0018 x 0.1347	(2', 4', 7', 8'): 0.0012 x 0.0779	(5', 7', 9', 9'): 0.0011 x 0.0643
(4', 8'): 0.0007 x 0.098	(1', 2', 5'): 0.0068 x 0.091	(3', 4', 7'): 0.0015 x 0.0335	(0', 1', 4', 8'): 0.003 x 0.0919	(0', 4', 5', 7'): 0.0011 x 0.0562	(1', 3', 6', 7'): 0.0011 x 0.0589	(2', 4', 8', 9'): 0.0013 x 0.0803	(5', 7', 9', 9'): 0.0011 x 0.0636
(5', 6'): 0.0013 x 0.1833	(1', 2', 6'): 0.0041 x 0.0743	(3', 4', 8'): 0.0016 x 0.0401	(0', 1', 4', 9'): 0.0033 x 0.2217	(0', 4', 5', 8'): 0.0017 x 0.1068	(1', 3', 6', 8'): 0.0013 x 0.0649	(2', 5', 6', 7'): 0.0011 x 0.0714	(5', 7', 9', 9'): 0.0011 x 0.0636
(5', 7'): 0.0008 x 0.106	(1', 2', 7'): 0.0042 x 0.0696	(3', 4', 9'): 0.0018 x 0.0325	(0', 1', 5', 5'): 0.0022 x 0.0922	(0', 4', 5', 9'): 0.0013 x 0.0922	(1', 3', 6', 9'): 0.0016 x 0.1191	(2', 5', 6', 8'): 0.0015 x 0.0714	(5', 7', 9', 9'): 0.0011 x 0.0636
(5', 8'): 0.0009 x 0.1223	(1', 2', 8'): 0.0056 x 0.071	(3', 5', 5'): 0.0018 x 0.0376	(0', 1', 5', 6'): 0.0021 x 0.0916	(0', 4', 6', 7'): 0.0011 x 0.0513	(1', 3', 7', 8'): 0.0013 x 0.0686	(2', 5', 6', 9'): 0.0014 x 0.0985	
(5', 9'): 0.0008 x 0.1088	(1', 2', 9'): 0.0097 x 0.1922	(3', 5', 6'): 0.0016 x 0.0376	(0', 1', 5', 7'): 0.003 x 0.0881	(0', 4', 6', 8'): 0.0016 x 0.1335	(1', 3', 7', 9'): 0.002 x 0.1824		

≈ 0.2047

41

41

Bayes Leakage Algorithm

1. Find distribution on secrets
2. Pick most likely secret $\leftarrow$ prior vulnerability
3. For all outputs:
 1. Isolate secrets that could produce the output
 2. Sum secrets' probabilities $\leftarrow$ prob. of output (PO)
 3. Normalize probabilities
 4. Pick most likely secret $\leftarrow$ prob. of success (PS)
4. $\Sigma(\text{PO} \times \text{PS})$ for all outputs $\leftarrow$ posterior vulnerability
5. Leakage: prior vs. posterior (+/x)

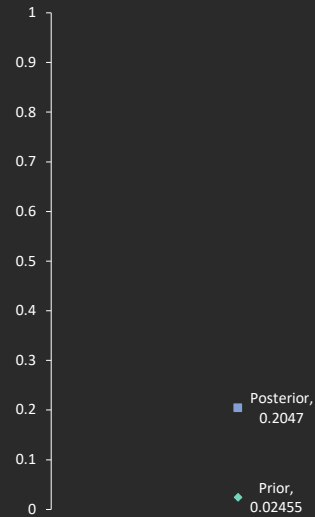
42

42

5. Leakage

- Multiplicative leakage = $\text{post} / \text{prior} = 8.3x$
- Additive leakage = $\text{post} - \text{prior} = 0.18$
- Summary:
 - Using the button keypad moves your vulnerability up to 20%, a 1 in 5 chance
 - Pay the extra \$30 for the touchscreen

VULNERABILITY



43

43

Order-Revealing Encryption

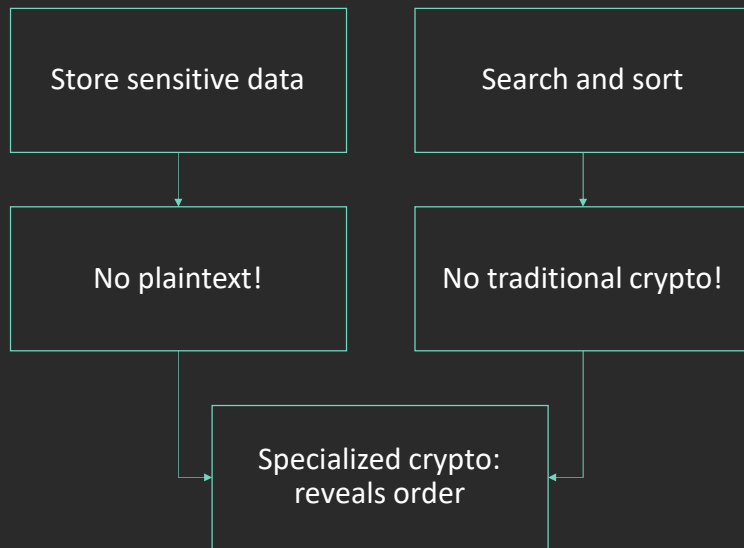
A Formal Information-Theoretic Leakage Analysis of Order-Revealing Encryption
 Mireya Jurado, Catuscia Palamidessi, Geoffrey Smith
 34th IEEE Computer Security Foundations Symposium (CSF21)



44

44

Motivation



45

45

Order-Revealing Encryption (ORE)

Insert

```
INSERT INTO employees (lname, fname, salary)
VALUES (Garcia, Maria, 28);
```

```
INSERT INTO employees (lname, fname, salary)
VALUES (Garcia, Maria, N6UD2uA);
```



DB admin



Proxy



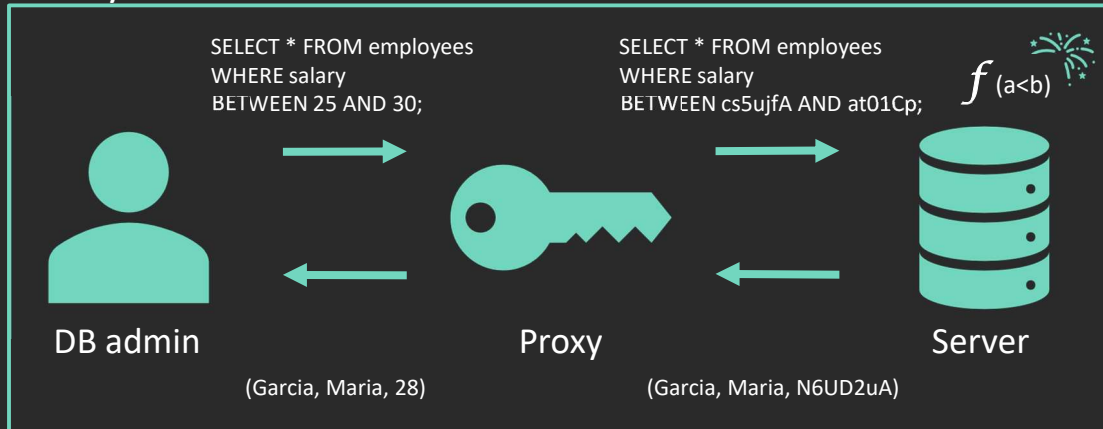
Server

46

46

Order-Revealing Encryption (ORE)

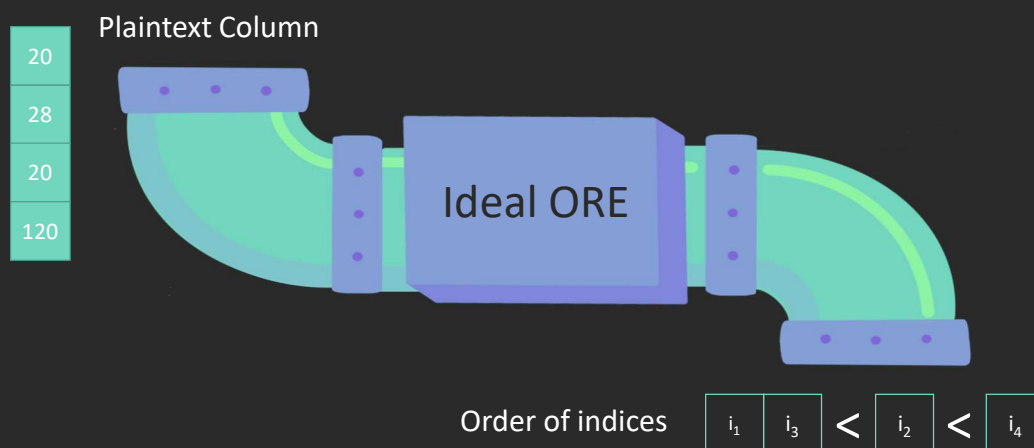
Query



47

47

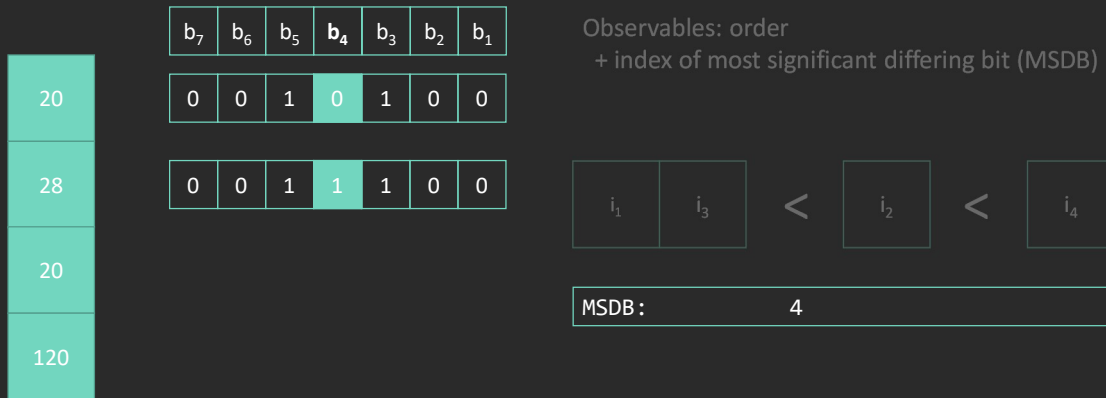
Ideal ORE



48

48

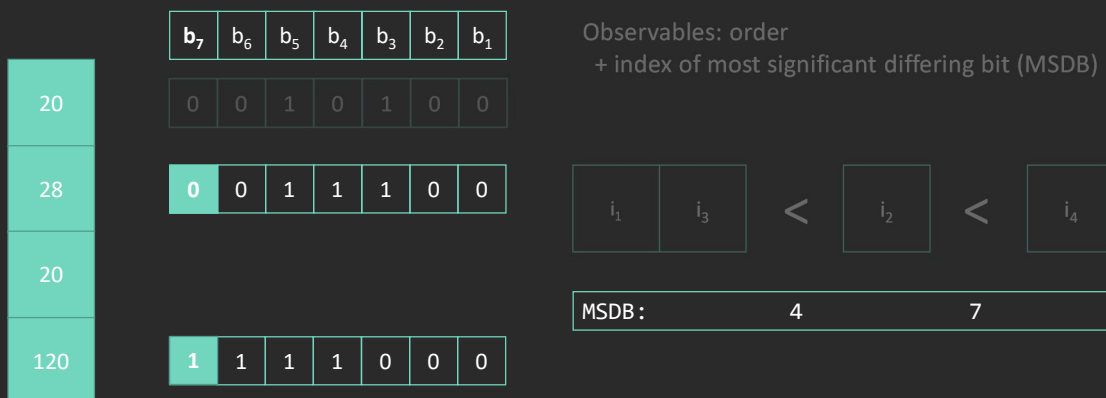
CLWW ORE

Chenette, Lewi, Weis, & Wu, Practical Order-Revealing Encryption with limited leakage. *Fast Software Encryption* (2016)

49

49

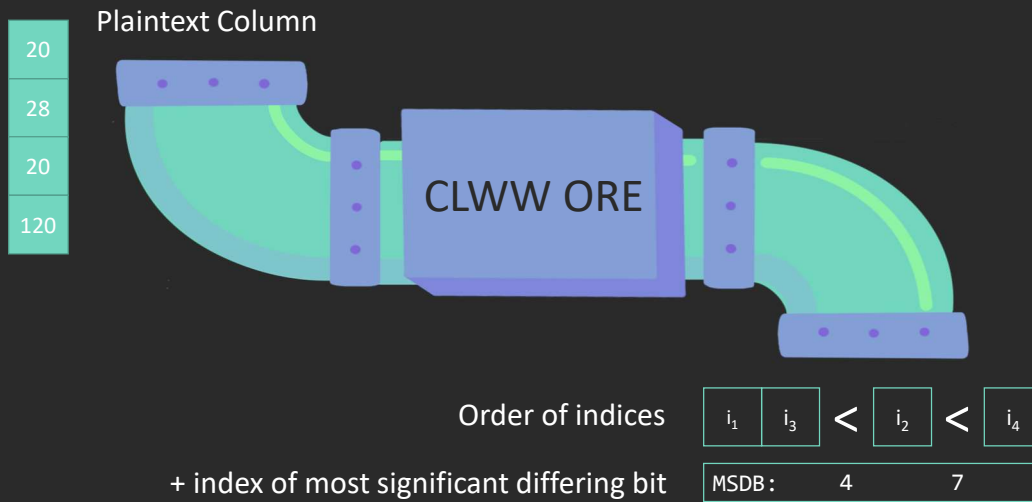
CLWW ORE

Chenette, Lewi, Weis, & Wu, Practical Order-Revealing Encryption with limited leakage. *Fast Software Encryption* (2016)

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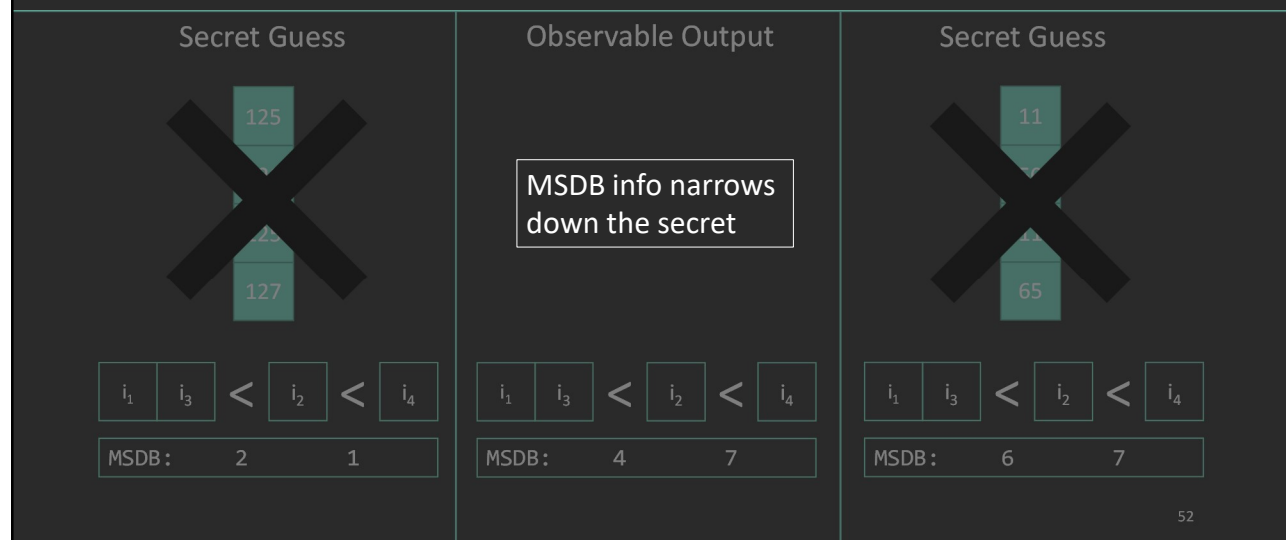
CLWW ORE



51

51

MSDB Intuition

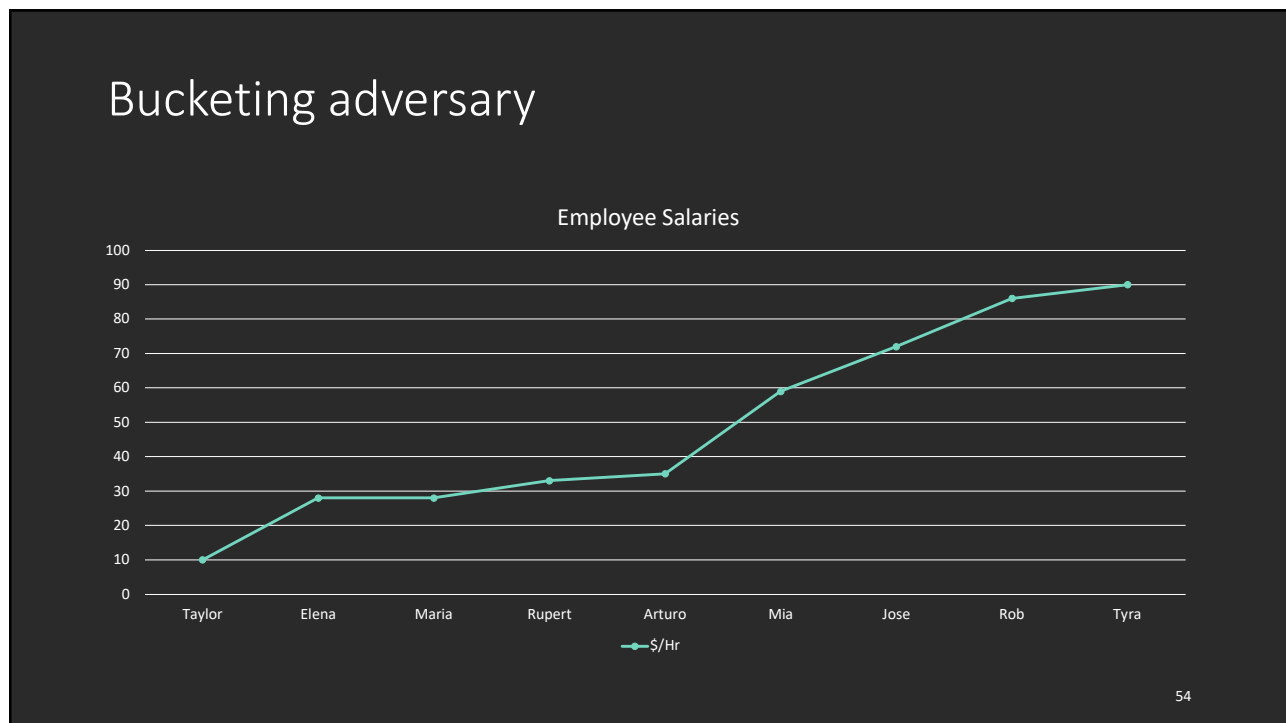


52

52



53



54

Bucketing adversary



Taylor

Elena

Maria



Rupert

Arturo

Mia



Jose

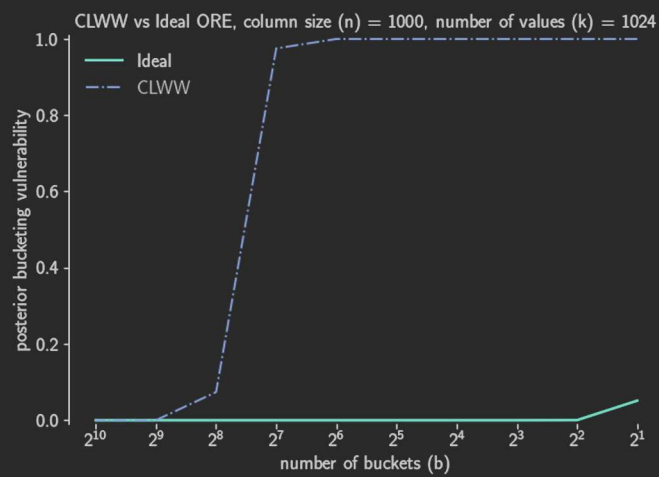
Rob

Tyra

55

55

Bucketing Vulnerability



56

56

Call to Action

Apply QIF:

- Quantify leakage exactly

Apply QIF principles:

- ID secret
- ID observables
- ID adversary
- Approximate distribution on secrets

Preventative Measure:

- Disassociate observables from secrets

Resources:

- Book: The Science of Quantitative Information Flow (2020)
- Blog: sprayonsecurity.com

57